Alternating Current Circuits Solution

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Full solution manual of Alternating Current Circuits by Russel M Kerchnel & George F Corcoran.

E-Book Written By

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Chapter–1

Newtwork Concepts

1.1 Given

There branch voltages are, $v_{b1} = (-2 + 3i_{b2})$

$v_{b2} = (-4 + 2i_{b2})$

$v_{b3} = 2i_{b3}$

(a) Using mesh analysis (mesh – 1)

$i_2 (2 + 2) + i_1 (2) = 4$

or, $i_2 (4) + i_1 (2) = 4$ ... ... (i)

Using mesh analysis, (mesh – 2)

$i_1 (2 + 2 + 1) + i_2 (2) = 2$

or, $5i_1 + 2i_2 = 2$ ... ... (ii)

If i_{b1}, i_{b2}, i_{b3} are the independent variable.

$i_2 = i_{b2}$

$i_{b3} = i_1 + i_2$ or, $i_2 = i_{b3} - i_{b1}$

$i_{b1} = i_1$

So equation (i) and (ii) can be written as,

$2i_2 + i_1 = 2$

or, $2i_{b2} + i_{b1} = 2$

and $5i_1 + 2i_2 = 2$

or, $5i_{b1} + 2i_{b2} = 2$

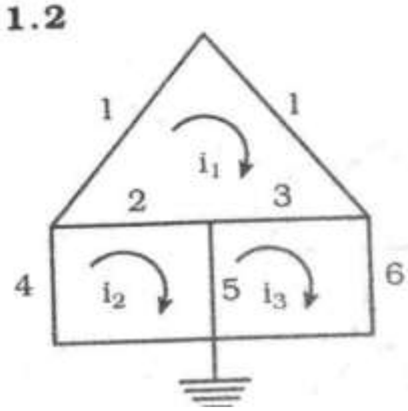
Thus,

$i_{b3} = i_1 + i_2$

$2i_{b2} + i_{b1} = 2$

$5i_{b1} + 2i_{b2} = 2$

1.2



Given, branch resistance,

$R_{b1} = 2$, $R_{b2} = 2$, $R_{b3} = 3$, $R_{b4} = 4$

$R_{b5} = 5$, $R_{b6} = 6$ ohms

Using mesh analysis.

Mesh-1

$$i_1(2+2+2+3)+i_2(-2)+i_3(-3)=E_1$$

or, $i_1(9)+i_2(-2)+i_3(-3)+e_{s1}+e_{s2}+e_{s3}$

Mesh-2

$$i_1(-2)+i_2(4+5+2)+i_3(-5)=E_2$$

or, $i_1(-2)+i_2(11)+i_3(-3)=e_{s4}+e_{s5}-e_{s2}$

Mesh-3

$$i_1(-3)+i_2(-5)+i_3(3+5+6)=E_3$$

or, $i_1(-3)+i_2(-5)+i_3(14)=e_{s6}-e_{s5}-e_{s3}$

Thus,

$$i_1(9)+i_2(-2)+i_3(-3)=e_{s1}+e_{s2}+e_{s3}$$

$$i_1(-2)+i_2(11)+i_3(-5)=e_{s4}+e_{s5}-e_{s2}$$

and, $i_1(-3)+i_2(-5)+i_3(14)=e_{s6}-e_{s5}-e_{s3}$

Number of branch and node (total).

$$b=37$$

and $n_t=4$

where,

$$n=(n_t-1)$$

$$=(4-1)=3$$

and $l=3$ independently.

(b)

Using mesh analysis.

Mesh-1

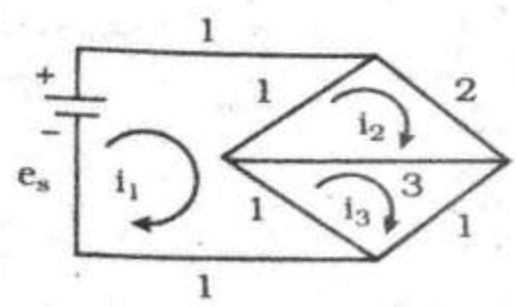
$$i_1(1+1+1+1)+i_2(-1)+i_3(-1)=e_s$$

Mesh-2

$$i_1(-1)+i_2(1+2+3)+i_3(-3)=0$$

Mesh-3

$$i_1(-1)+i_2(-3)+i_3(3+1+1)=0$$



Simplifying.

$i_1(4) + i_2(-1) + i_3(-1) = e_5$

$i_1(-1) + i_2(6) + i_3(-3) = 0$

and. $i_1(-1) + i_2(-3) + i_3(5) = 0$

(c) $$i_1 = \frac{\begin{vmatrix} e_5 & -1 & -1 \\ 0 & 6 & -3 \\ 0 & -3 & 5 \end{vmatrix}}{\begin{vmatrix} 4 & -1 & -1 \\ -1 & 6 & -3 \\ -1 & -3 & 5 \end{vmatrix}} = \frac{e_s(30-9)}{67} = \frac{e_s 21}{67}$$

$\therefore \quad \frac{i_1}{e_5}$ or i_1 per $e_5 = \frac{21}{67}$

$i_1 = -2.5$

$i_2 = -1$

$i_3 = -1.07$

So, $i_{3r} = i_2 - i_3$

$= 0.07$ amp

1.4

(a) Mesh–1

$i_1(1 + 2 + 1 + 1) + i_2(-2) + i_3(-1) = 8$

or, $i_1(5) + i_2(-2) + i_3(-1) = 0$

Mesh–2

$i_1(-2) + i_2(1 + 2 + 3) + i_3(-3) = 0$

or, $i_1(-2) + i_2(6) + i_3(-3) = 0$

Mesh–3

$i_1(-1) + i_2(-3) + i_3(3 + 1 + 1) = 0$

or, $i_1(-1) + i_2(-3) + i_3(5) = 0$

Thus,

$i_1(5) + i_2(-2) + i_3(-1) = 8$

$i_1(-2) + i_2(6) + i_3(-3) = 0$

$i_1(-1) + i_2(-3) + i_3(5) = 0$

So, $i_1 = +2.5$ amp

$i_2 = +1.56$ amp

$i_3 = +1.43$ amp

$\therefore$ total wrrent $i_1 = +2.5$ amp

(b) Power delivered = $8 \times 2.5 = 20$ amp

(c) $i_{3\Omega} = 1.43 - 1.56$
$= +0.13$ amp

1.5

Here, $b = 8$
$n_t = 5$
$n = n_t - 1 = 4$
and $l = 3$

(b) Number of unknown = number of equation

To make this above equation correct, sufficient data must be given.

(c) $m_1 \rightarrow i_1 (1 + 1 + 1) + i_2 (-1) + i_3 (-1) = 0$ [m represents mesh]
$m_2 \rightarrow i_1 (-1) + i_2 (2 + 1 + 1 + 1) + i_3 (1) = 0$
$m_3 \rightarrow i_1 (-1) + i_2 (1) + i_3 (1 + 1 + 1 + 1) = 0$

Thus,
$i_1 (3) + i_2 (-1) + i_3 (-1) = 0$
$i_1 (-1) + i_2 (5) + i_3 (1) = 0$
and, $i_1 (-1) + i_2 (1) + i_3 (4) = 0$

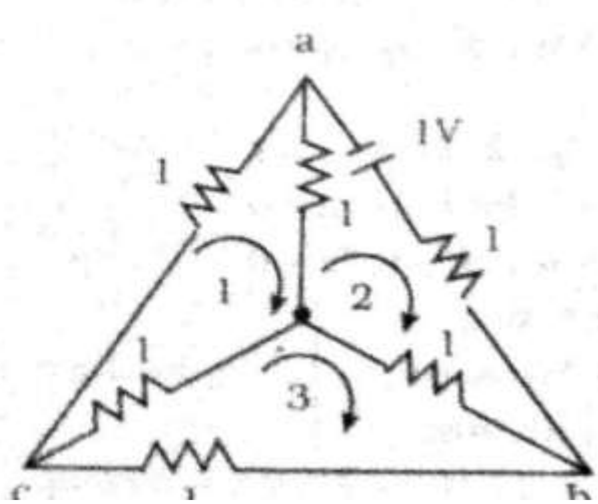
$$R_{matrix} = \begin{vmatrix} 3 & -1 & -1 \\ -1 & 5 & 1 \\ -1 & 1 & 4 \end{vmatrix}$$

determinant of $R_{matrix} = 50$

1.6 Draw yourself.

1.7 Draw Yoursef.

1.8



Mesh–1

(a) $i_1 (1 + 1 + 1) + i_2 (-1) + i_3 (-1) = 0$

Mesh–2

$i_1 (-1) + i_2 (1 + 1 + 1) + i_3 (-1) = 1$

Mesh–3

$i_1 (-1) + i_2 (-1) + i_3 (1 + 1 + 1) = 0$

Thus, $i_1 (3) + i_2 (-1) + i_3 (-1) = 0$

$i_1 (-1) + i_2 (3) + i_3 (-1) = 1$

and, $i_1 (-1) + i_2 (-1) + i_3 (1 + 1 + 1) = 0$

$i_1 = 0.25$

$i_2 = 0.5$

$i_3 = 0.25$

(b) The answer is intuitive.

1.9 Mesh–1

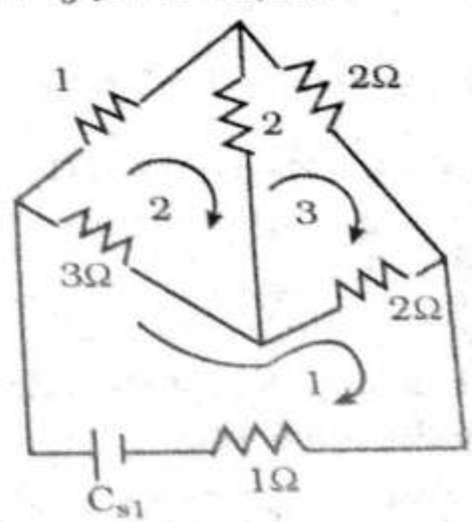
(a) $i_1 (1 + 2 + 3) + i_2 (-3) + i_3 (-2) = e_{s1}$

Mesh–2

$i_1 (-3) + i_2 (1 + 2 + 3) + i_3 (-2) = 0$

Mesh–3

$i_1 (-2) + i_2 (-2) + i_3 (2 + 2 + 2) = 0$



Thus,

$i_1 (6) + i_2 (-3) + i_3 (-2) = e_{s1}$

$i_1 (-3) + i_2 (6) + i_3 (-2) = 0$

and $i_1 (-2) + i_2 (-2) + i_3 (6) = 0$

(b) $R_{mat} = \begin{vmatrix} 6 & -3 & -2 \\ -3 & 6 & -2 \\ -2 & -2 & 6 \end{vmatrix} = 90$

1.10 Mesh-1

$i_1 (1 + 2 + 2 + 1) + i_2 (-1) + i_3 (-2) = e_{s1}$

Mesh-2

$i_1 (-1) + i_2 (3 + 2 + 2 + 1) + i_3 (2) = e_{s1}$

Mesh-3

$i_1 (-3) + i_2 (2) + i_3 (1 + 2 + 3 + 2) = 0$

Thus,

$i_1 (6) + i_2 (-1) + i_3 (-2) = e_{s1}$

$i_1 (-1) + i_2 (8) + i_3 (2) = e_{s1}$

and $i_1 (-3) + i_2 (2) + i_3 (8) = 0$

11. (a)

Mesh-1

$i_1 (1 + 1 + 1 + 1) + i_2 (1 + 1 + 1) + i_3 (1 + 1) + i_4 (1 + 1) =$
$-e_{s1} + e_{s2} = e_{s2} - e_{s1}$

or, $i_1 (4) + i_2 (2) + i_3 (2) + i_4 (2) = e_{s2} - e_{s1}$

Mesh-2

$i_1 (1 + 1) + i_2 (1 + 1 + 1) + i_3 (-1) + i_4 (-1) = e_{s2}$

Mesh-3

$i_1 (1 + 1) + i_2 (-1) + i_3 (1 + 1 + 1 + 1) + i_4 (1) = 0$

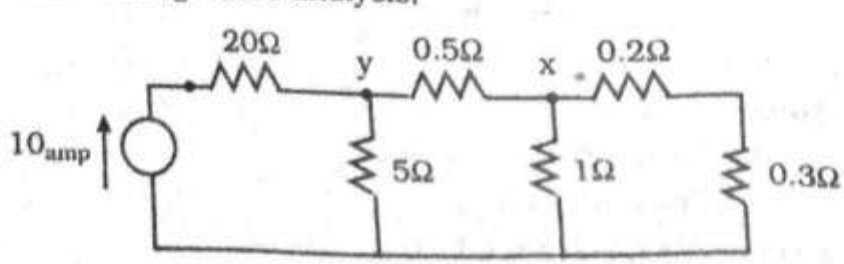
Mesh-4

$i_1 (1 + 1) + + i_2 (-1) + i_3 (1) + i_4 (1 + 1 + 1) = -e_{s1}$

(b) $R_{matrix} = \begin{vmatrix} 4 & 2 & 2 & 2 \\ 2 & 3 & -1 & -1 \\ 2 & -1 & 4 & 1 \\ 2 & -1 & 1 & 3 \end{vmatrix}$

$R_{determinat}$ = do yourself (use + three independent equation)

1.12 Using Nodal analysis,



Node X,

$$v_x\left(\frac{1}{0.5}+\frac{1}{1}+\frac{1}{0.5}\right)+v_y\left(-\frac{1}{0.5}\right)=0$$

Node Y,

$$v_x\left(-\frac{1}{0.5}\right)+v_y\left(\frac{1}{5}\right)=+10$$

So, (a) $v_x = -6.67$ volt

(b) $v_y = -16.67$ volt

1.13

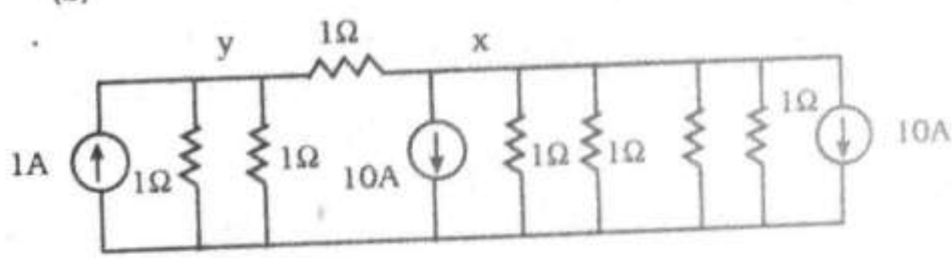
$b = 6$

$n_t = 3$

$n = n_t - 1 = 2$

$\therefore \quad l = b - n = 4$

(b)



1.14

Node X

$v_x(1+1+1)+v_y(-1)+=-1-10=-11$

or, $v_x(3)+v_y(-1)$

Node Y

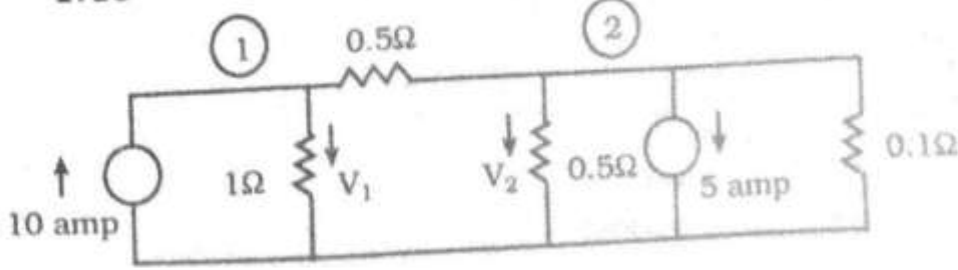
$v_x(-1)+v_y(1+1+1)=+1$

or, $v_x(-1)++v_y(3)=1$

Thus, $v_x = -4$

$v_y = -1$

1.15



Nodal analysis :

Node–1

$v_1\,(1 + 0.5) + v_2\,(-0.5) = 10$

Node–2

$v_1\,(0.5) + v_2\,(0.5 + 0.5 + 0.1) = -5$

Thus. $v_1\,(1.5) + v_2\,(-0.5) = 10$

and $v_1\,(-0.5) + v_2\,(1.1) = -5$

so, $v_1 = 6.07$ volt

and $v_2 = -1.785$ volt

1.16 The answer is intuitive and similar to number 15.

1.17

from fig 30 (a)

$v_1' = v_2'$

$I = i_{11} = 1.6$ amp

$v_1' = v_2'$

$= 1.6 \times (1+0.6)$

$= (1.6)^2 = 2.5$ volt

from fig 30 (b)

$v_1'' - v_2'' = 1$ – (i)

$v_1''\,(1 + 0.5) + v_2''\,(-0.5)$

$= -0.6$

so, $v_1'' = -1.1$ volt

$v_2'' = -2.1$ volt

from fig 30 (c)

$v_1''' = 1v$

$v_1'''\,(-0.5) + v_2'''\,(0.5+0.6)$

$= -1_{21}$

$= 0.6$

$v_1'''\,(-0.5) + v_2'''\,(1.1)$

$v_2'''\,(1.1) = 1.1$

$v_2''' = 1_v$

from fig 30 (d) .

$v_1^{iv} = 0$ -1_{22}

$v_1^{iv}\,(-0.5) + v_2^{iv}\,(0.5+0.6)$

$= -1.1$

$v_2^{iv} = \frac{-1.1}{1.1} = -1$ vo

from fig 30 (e)

$v_1^{v} = v_2^{v} = 0$

from fig 30 (f)

$v_1^{vi} = v_2^{vi} = 0$

So, $v_1 = v_1' + v_1'' + v_1''' + v_1^{iv} + v_1^{v}$

$= 2.56 - 1.1 + 0 + 0$

$= 1.46$ volt

$v_2 = v_2' + v_2'' + v_2''' + v_2^{iv} + v_2^{v}$

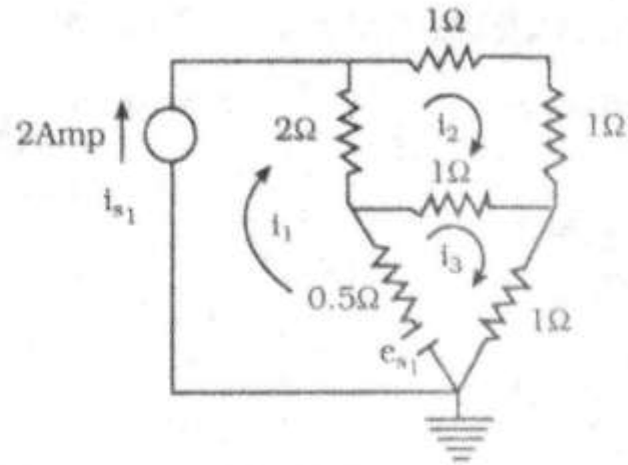
$= 2.56 - 2.1 + 1 + 0 + 0$

$= 1.46$ volt

1.18 Draw yourself

1.20 Draw yourself

1.21



Mesh-1

$i_1 = 2\text{amp}$

Mesh-2

$i_1(-2) + i_2(2 + 1 + 1) + i_3(-1\Omega) = +e_{s2}$

or, $i_1(-2) + i_2(4) + i_3(-1) = 2$

Mesh-3

$i_1(-0.5) + i_2(-1) + + i_3(1 + 0.5) = -e_{s2}$

Thus,

$i_1(-2) + i_2(4) + i_3(-1) = 2$

or, $i_2(4) + i_3(-1) = 6$

and $i_1(-0.5) + i_2(-1) + i_3(1.5) = -2$

$i_2\ (-1) + i_3(1.5) = -1$

so, $i_2 = 1.6$ A and $i_3 = 0.4$A

$V_a = (1 + 1)\, i_2$

$= 2 \times 1.6$

$= 3.2$ Volts.

(b)

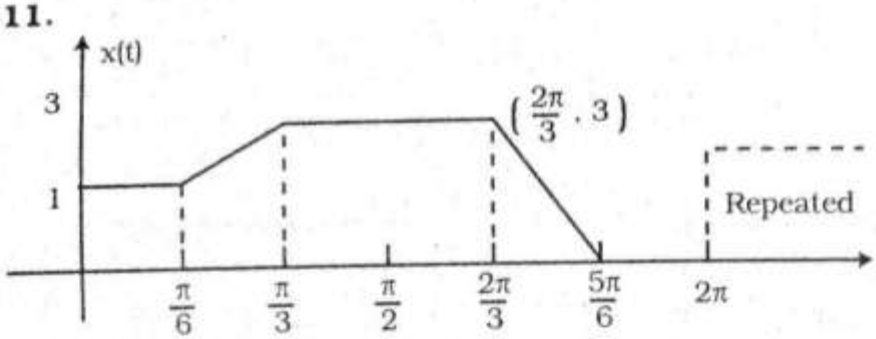
$$= \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$$= \sum_{n=-\infty}^{\infty} \left[\frac{5}{-j3\pi}(e^{-j3\pi} - 1) + \left(\frac{-5}{2\pi}\right) \times \frac{1}{-3j}(e^{-6\pi j} - e^{j3\pi})\right] \times e^{jnt}$$

(c) Do yourself

6.10 (a) Same to number 9.

6.11.



$$c_n = \frac{1}{2\pi}\int_0^{2\pi} x(t)e^{-jnt}dt$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi/6} e^{-jnt}\,dt + \int_{\pi/6}^{\pi/3}\left(\frac{2}{\pi/3}t\right) e^{-jnt}\,dt + \int_{\pi/3}^{2\pi/3} 3e^{-jnt}\,dt\right.$$

$$+ \int_{2\pi/3}^{5\pi/6}\left[\frac{18}{\pi}(t - 2\pi/3) + 3\right]dt$$

As these integrations are easier to do and can be evaluated by by parts formula, try yourself.

6.12 Not important

6.13 Not important

6.14

$$V = 4\sin\omega t - 3\cos\omega t - 7.66\sin 2\omega t + 6.43\cos 2\omega t$$
$$- 2\sin 3\omega t - 1.5\cos 3\omega t$$

$$= \sqrt{4^2 + 3^2}\sin(\omega t - \tan^{-1} 4/3) - \sqrt{(7.66)^2 + (6.43)^2}$$
$$\sin(2\omega t - \tan^{-1}\frac{7.66}{6.43}) - \sqrt{2^2 + 1.5^2}\sin(3\omega t + \tan^{-1} 2/1.5)$$

$$= 5\sin(\omega t - 53.13) - 10\sin(2\omega t - 49.98)$$
$$- 2.5\sin(3\omega t + 53.13)$$

Chapter–2

Instantaneous Current, Voltage, and Power

2.1

Here given,

Pole $P = 10$

$N = 360$ rpm

(a) Frequency,

$$f = \frac{PN}{120} = \frac{10 \times 360}{120}$$

$$= 30 \text{ Hz}$$

(b) Here,

$$f = \frac{PN}{120}$$

$$N = \frac{120f}{P}$$

For $f = 25$ Hz
$N = 500$ rpm

$f = 30$ Hz
$N = 60$ rpm

$f = 50$ Hz
$N = 1000$ rpm

$f = 60$ Hz
$N = 1200$ rpm

2.2 Here,

$$f = \frac{PN}{120}$$

$$P = \frac{120\,f}{N}$$

$$= \frac{120 \times 50}{300}$$

$$= 20 \text{ poles}$$

2.3 Here,

$$\omega_e = 2\pi f$$

$$= 314.159 \text{ rad/sec}$$

This is the electrical angular velocity.

$$\omega_m = 2\pi \times \frac{N}{60}$$

$$= 31.41 \text{ rad/sec}$$

2.4 Here,

$$i = i_m \sin(\omega t + \theta)$$

where, at $t = 0$, $i = 1.5$A, $i_{max} = 1.732$ and $f = 1591$ KHz

so, $1.5 = 1.732 \sin(2\pi \times 1591 \times 10^3 \times 0 + \theta)$

$$= 1.732 \sin\theta$$

$$\therefore \quad \sin\theta = \frac{1.5}{1.732} = 0.866$$

$$\theta = 60.0029 \approx 60^\circ$$

2.5 If we exprees the sine wave decribed in the question.

$$i = i_{max} \sin(2\pi \times 50 \times t + \theta)$$

where, $i_{max} = 10$ amp

so, $i = 10 \sin(100\pi t + \theta)$

(i) Angular velocity $\omega = 2\pi f = 2\pi \times 50 = 100\pi$ rad/sec

2.6 If we express the sine wave described in the question.

$$i = i_{max} \sin(2\pi ft + \theta)$$

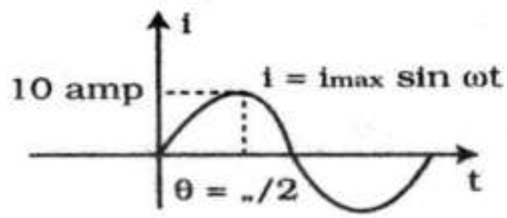
$$= 10 \sin(2\pi \times ft + \theta) = 10 \sin(377t + \theta)$$

where, $\omega = 377$ rad/s and $i_{max} = 10$ amp

frequency $f = \frac{\omega}{2\pi} = \frac{377}{2\pi} = 60.0014 \approx 60$ Hz

2.7

(a)



letting, $\theta = 0°$

so, $i = i_{max} \sin(\omega t - \frac{\pi}{2})$

$= 10 \sin(\omega t - \frac{\pi}{2})$.

(b) Here,

$$i = 10 \sin(\omega t + \theta)$$

$$-5 = 10 \sin(314.15t + \theta)$$

so, $\sin\theta = -0.5$

$$\theta = -30 = -\frac{\pi}{6}$$

$\therefore \quad i = 10 \sin(314.15t - \pi/6)$

2.8 Given,

$$e = 100 \sin 157t \text{ volt}$$

(a) maximum value = 10

(b) frequency $= \frac{157}{2\pi} = 25$ Hz

(c) for $e = 100 \sin(157t + 130)$

maximum value = 100

frequency $= \frac{157}{2\pi} = 25$ Hz

2.9 Page 51, oscillogram 1

$T = \frac{1}{60}$ and rate of change of voltage $= \frac{dv}{dt}$

Here,

$v = -140 \sin (377t)$

so, $\frac{dv}{dt} = -140 \times 377$ 60s $377t$

$= -52780 \cos 377t$

So the maximum ROC of voltage = 52780

Thus the minimum ROC of voltage = –52780

2.10 Given,

$i = 10 \sin (\omega t - 30)$ ... ... (i)

so, $\frac{di}{dt} = 10 \times \omega \cos (\omega t - 30)$

at t = t, $3265 = 10 \times 377 \cos (377t - 30)$

or, $30^\circ = (377t)^c - 30^\circ$ and $\omega t = 60^\circ$

or, $\frac{\pi}{6} = 377t - \frac{\pi}{6}$

or, $t = 2.77 \times 10^{-3}$

From equation (i)

$i = 10 \sin (60^\circ - 30^\circ)$

$= 5$ amp

(b) for $i = 2000$ amp

$i = i_{max} \sin (\omega t - 30^\circ)$

or, $\frac{di}{dt} = i_{max} \times \omega \cos (\omega t - 30^\circ)$

or, $2000 = 10 \times 377 \cos (377t - 30^\circ)$

or, $\cos (377t - 30^\circ) = 0.599$

or, $377t - 30^\circ = 53.167^\circ$

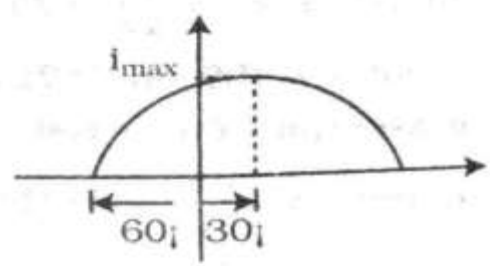
so, $i = 10 \sin 53.167 = 8$ amp

2.11

$i = I_m \sin (\omega t + \theta)$

$\frac{di}{dt} = I_m \frac{d}{dt} \sin (\omega t + \theta)$

$= I_m \times \omega \cos (\omega t + \theta)$



$\frac{dt}{dt} = I_m \times 2\pi \times 50 \cos (2\pi \times 50 \times t + \theta)$

Where at $t = 30^\circ$ phase shift of value gero.

$$\frac{di}{dt} = 2000$$

$$\theta = 30^\circ$$

so, $2000 = I_m \times 2\pi \times 50 \cos(2\pi \times 50 \times 0 + 30^\circ)$

or, $2000 = I_m \times 315 \cos 30^\circ$

or, $I_m = 7.33$ amp

2.12

$v = 100 \sin(\omega t - 30^\circ)$ phase $= \omega t - 30^\circ$

and $i = 10 \sin(\omega t - 60^\circ)$ phase $= \omega t - 60^\circ$

$\therefore$ phase difference $= \omega t - 30^\circ - \omega t + 60^\circ$

$= 30^\circ$

[Tips, take voltage as reference o vice versa.

So the voltage leads the current by 30°.

2.13

$v = 100 \cos(\omega t - 30^\circ)$

$i = 10 \sin(\omega t - 60^\circ)$

Here,

$$Q_v - Q_i = \omega t - 30^\circ + \omega t + 60^\circ$$
$$= 30^\circ$$

so v leads i or i logs v.

2.14 Given,

voltage $v = 100 \cos \omega t$

$i_{max} = 10$ amp

A cycle $= 2\pi$ radian

$\frac{1}{6}$ cycle $= \frac{\pi}{3}$ radian

[Tips : cos shoul reference]

so, $i = i_{max} \cos(\omega t + 60^\circ)$

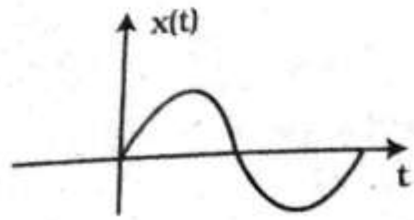
$= 10 \cos(\omega t + 60^\circ)$

2.15 (a) $x(t) = K \sin(2\pi ft)$

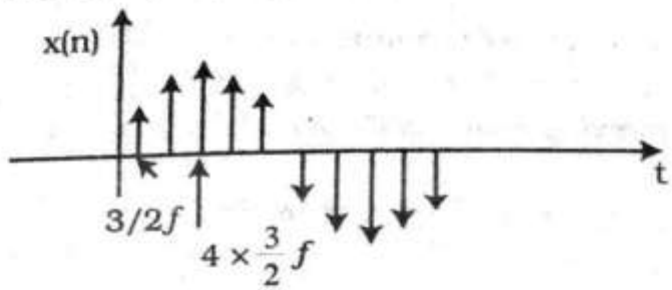
$$f_s = 2\frac{f}{3}$$

Then the sampled signal has perio

discrete signal.

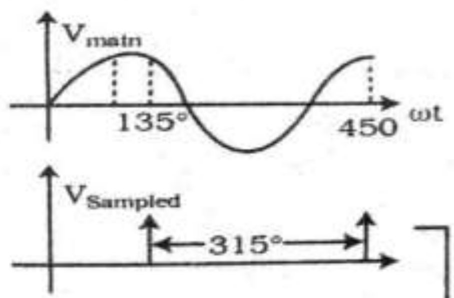


After sampling (approximately)



(b) Do yourself.

[Tips : For one cycle



]

2.16

Given,

$I_{max} = 90$ amp

$\omega t_1 = 60°$

$= \frac{\pi}{3}$

Finding instantaneous value of this sinusoidal a.c.,

$i = 90 \sin 60°$

$= 90 \times \frac{\sqrt{3}}{2} = 77.94$ amp

or, $t_1 = \frac{\pi}{3 \times 2\pi \times 50}$

$= 0.0033$ sec

for $\omega t_2 = 225°$

or, $t_2 = \frac{225 \times \pi}{180 \times 2\pi \times 50} = 0.0125$

$\therefore \quad t_2 - t_1 = 0.0125 - 0.033 = 0.0092$ sec

Ans : The difference between 60° and 225° is –0.0092 sec.

2.17

Given,

$$i = 1 + 0.50 \sin 1885t - 0.1 \cos 3770t \text{ amp}$$

or $i = 1.0 + 0.5 \sin\alpha - 0.1 \cos 2\alpha$ amp

where $\alpha = 1885t$ radians if t is expreesed in seconds.

(a) Sine wave :

$\omega = 2\pi f$

$1885 = 2\pi f$

$f = 300$ Hz

Cosine wave :

$\omega = 2\pi f$

$3770 = 2\pi f$

$f = 600$ Hz

(b) When,

$\sin\alpha = 1$

$\therefore \quad \alpha = \frac{\pi}{2}$

we have,

$$i = 1.0 + 0.5 \sin\frac{\pi}{2} - 0.1 \cos\pi = 1.6 \text{ amp}$$

when,

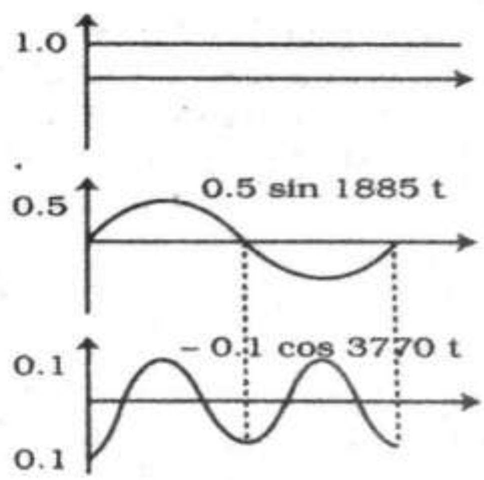
$\sin\alpha = -1$

$\therefore \quad \alpha = -\frac{\pi}{2}$

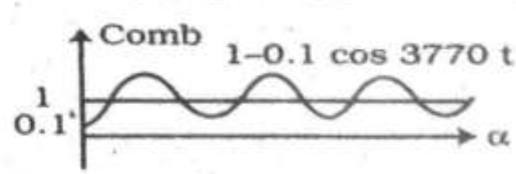
we have,

$$i = 1 + 0.50 \sin\left(-\frac{\pi}{2}\right) - 0.1 \cos(-\pi) = 0.6 \text{ amp}$$

(c)



Combining graph. (approximately)



2.18 Given,

$v = 150 \cos\ 314t$ and $R = 30$ ohms

(a) $i = \frac{150}{30} \cos 314t = 5 \cos 314t$

(b) $f = \frac{\omega}{2\pi} = \frac{314}{2\pi} = 50$ Hz

(c) $P = vi = 150 \cos 314t \times 50 \cos 314t$

$= 750 \cos^2 314t$

$= \frac{1}{2} (1 + \cos 2 \times 314t) \times 750$ [Tips : Simplification is important in EEE]

$= 375 + 375 \cos 628t$ walts

(d) Frequency of power variation,

$f = \frac{\omega}{2\pi} = \frac{628}{2\pi} = 100$ Hz

2.19 Given,

$i = 5 \sin (110t + 30°)$ and $R = 20\ \Omega$

(a) $v = 5 \sin (110t + 30) \times 20 = 100 \sin (110t + 30°)$ v

(b) $f_v = \frac{110}{2\pi} = 17.5$ Hz

(c) $P = vi = 100 \sin (110t + 30°) \times 5 \sin (110t + 30°)$

$= 500 \sin_2 (110t + 30°)$

$= 250 \times 2 \sin^2 (110t + 30°)$

$= 250 \times [1 - \cos 2 (110t + 30°)]$

$= 250 \times [1 - \cos (220t + 60)]$

$= 250 - 250 \cos (220t + 60)$

(d) $f_p = \frac{220}{2\pi} = \frac{220}{2\pi} = 35$ Hz

2.20 Given,

$v = 100 \cos (\omega t + 60°)$, $R = 10\Omega$

(a) $i = \frac{100}{10} \cos (\omega t + 60°) = 10 \cos (\omega t + 60°)$

(b) $p = vi = 100 \cos (\omega t + 60) \times 10 \cos (\omega t + 60°)$

$= 1000 \cos^2 (\omega t + 60)$

$= 500 \times 2 \cos^2 (\omega t + 60)$

$= 500 \times [1 + \cos 2 (\omega t + 60)]$

$= 500 + 500 \cos (2\omega t + 120)$ watts

(c) Maximum power, when $\cos(2\omega t + 120) = 1$

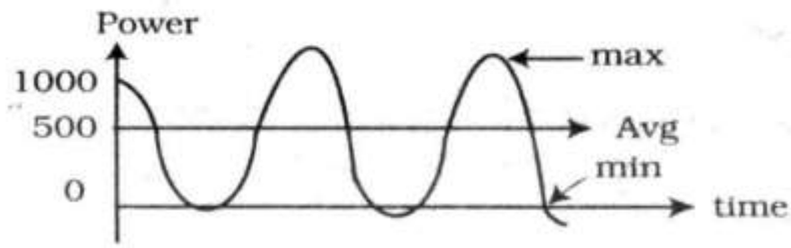
$\therefore$ $P_{max} = 500 + 500 = 1000$ watts

(d) Minimum power, when $\cos(2\omega t + 120°) = -1$

$P_{min} = 500 - 500 = 0$ watts.

(e) $P_{av} = \frac{v_m I_m}{2} \cos\theta = \frac{100 \times 10}{2} \cos\theta$ [where $\theta = 0°$ means phase shift $= 0$]

$= 500$ watts



2.21 Given, $i = 10 \sin 377t$

(a) $\frac{di}{dt} = 10 \times 377 \cos 377t$

$\therefore$ maximum value $= 10 \times 377 = 3770$ amp

when, $\cos 377t = 1$

(b) $x_1 = \omega L$ where inductance $L = 100$ mH

$= 377 \times 100 \times 10^{-3} = 37.7\ \Omega$

$\therefore$ $v_{max} = I_{max}\, x_L = 10 \times 37.7 = 377$

So voltage equation,

$v = 10 \times 37.7 \sin(377t + 90°) = 377 \sin(377t + 90°)$

where, $v_{max} = 377$ volt

2.22 Given, $v = -150 \sin 377t = 150 \sin(377t + 180°)$

and $i = 10 \cos 377t = 10 \sin(377t + 90°)$

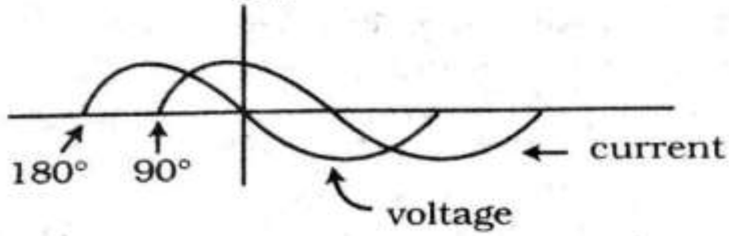
As voltage angle differes from the current angle $(180° - 90°)$ or $90°$

So voltage leads current by $90°$.

From the defination of inductive circuit, the impedence is inductive.

$\therefore$ impedence function, $\frac{150}{10} \angle 90 = 15 \angle 90$

Now, 377L = 15, $L = \frac{15}{377} = 0.0398$ henry



2.23 Given,

$v = 100 \sin (377t + 30°)$

$L = 0.025654$ H

so, $x_L = \omega_L = 377 \times 0.02654 = 10\ \Omega$

(a) Current, $v = ix_L$

$$i = \frac{v}{x_L} = \frac{100}{10} \sin (377t + 30° - 90°)$$

[From the defination, currents lage from voltage.]

$$= 10 \sin (377t - 60°)$$

(b) $P = v_i = (377t + 30°) \times 10 \sin (377t - 60°)$

$= - 1000 \sin (377t + 30°) \cos (377t - 60° + 90°)$

$= - 1000 \sin (377t + 30°) \cos (377t + 30°)$

$= - 500 \sin 2(377t + 30°)$

$= - 500 \sin (754t + 60°)$

(c) $P_{av} = -\frac{v_m I_m}{2} \cos \theta = \frac{100 \times 10}{2} \cos 90° = 0$ watts

(d) $P = - 500 \times 754 \cos (754t + 60°)$

finding differentiation,

$$\frac{dp}{dt} = - 500 \times 754 \cos (754t + 60°)$$

finding maximum value,

$$\left(\frac{dp}{dt}\right) = 0 \text{ so, } \cos (754t + 60°) = 0$$

(e) Energy stored $= \frac{v_m I_m}{2w} = \frac{100 \times 10}{2 \times 377} = 1.326$ joule

2.24 Given, $i = 5 \sin 300t$

(a) Impedance function is $300 \times 0.2 \angle 90° = 60 \angle 90°$

(b) $P = v_i$

where,

$v = 300 \sin (300t + 90°)$

$= 300 \cos 300t$ [c]

$= 300 \cos 500t \times 5 \sin 300t$

$= 1500 \sin 300t \cos 300t$

$= 750 \sin 600t$

Stored energy $(\omega_2) = \int_0^{0.05} 750 \sin 600t$

$= \frac{-750}{600} [\cos 600t]_0^{0.05}$

$= \frac{-750}{600}\left(\frac{\sqrt{3}}{2} - 1\right)$

$= 0.1675$ Joules

2.25 Given,

$v = 200 \cos (157t + 30°)$

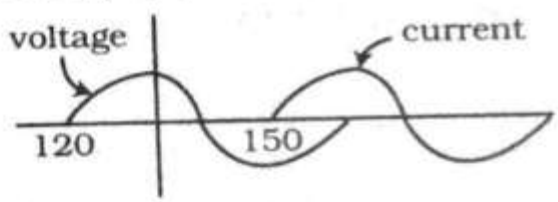
and $i = 5 \sin (157t - 150°)$

so, voltage $v = 200 \sin (157t + 30° + 90°)$

or, $v = 200 \sin (157t + 120°)$

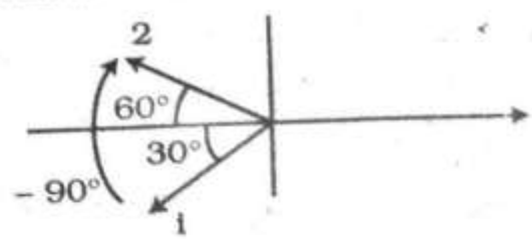
$i = 5 \sin (157t - 150°)$

Voltage is leading by $(120° - (-150°)$ or $270°$



Similarly it could be said as leading by −90°

Similarly it could be said as leading by 90°



$z = \frac{200}{5} \angle -90° = 40 \angle -90°$ ohms

The ciwit parameter is capacitor $x_c = \frac{1}{\omega c}$

or, $40 = \frac{1}{157c}$

$\therefore \quad c = 159.23 \times 10^{-6}$ F

$= 159.23\ \mu F$

2.26

$v = 100 \sin 377t$

$c = 530.\dot{5}\ \mu F$ and $\omega = 377$

$x_c = \frac{106}{377 \times 530.5\ \mu} = 50\Omega$

(a) $i = \frac{100}{5} \sin(377t + 90°) = 20 \sin(377t + 90°)$

(b) $P = vi = 100 \sin(377t) \times 20 \sin(377t + 90°)$

$= 1000 \sin 754t$ [using $\sin 2\theta = 2\sin\theta \cos\theta$]

(c) ω_c (energy max in capacitor)

$= \frac{v_m i_m}{2} = \frac{100 \times 20}{2 \times 377} = 2.65$ Joules

When the current is in maximum $I_m = 20$ amp

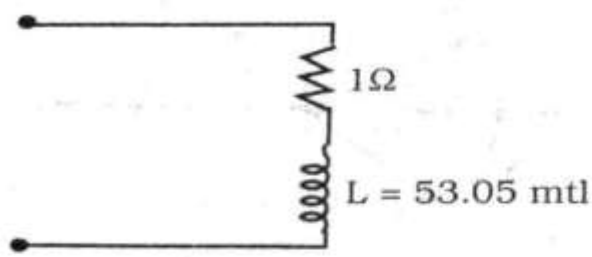
and $\omega_c = 0$ when the current $i_{min} = 0$

2.27

$v = 200 \sin 377t,\ x_L = \frac{200}{10} = 20$ ohm

(a) $L = \frac{x_L}{\omega} = \frac{20}{377} = 53.05$ mH

(b) $R_L = 20\ \Omega$

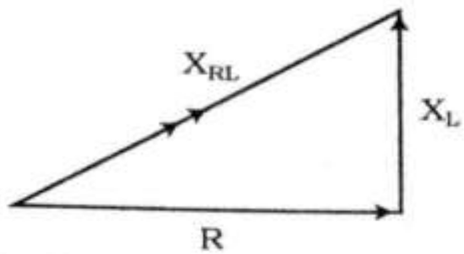


L = 53.05 mH

$X_{RL} = 20\ \Omega$

$X_L = \sqrt{20^2 - 1^2} = 19.975\ \Omega$

$L = \frac{X_L}{\omega} = 52.98$ mH



$$\begin{cases} x_{RL} = \sqrt{R^2 + x_L^2} \\ \theta_L = \tan^{-1} \frac{X_L}{R_L} \end{cases}$$

In phasor form,

$x_{RL} = R + x_L$

[Tips : You can do the sum, multi, div and solution by using calculator.]

2.28 Given,

$R = 10\ \Omega \quad x_L = 2\pi fL = 7.854\ \Omega \quad v_m = 150$ v

(a) $x_{RL} = \sqrt{R^2 + x_L^2} \angle\tan^{-1} \frac{X_L}{R}$

$= \sqrt{10^2 + 7.854^2} \quad \angle\tan^{-1} \frac{7.854}{10}$

$= 12.71 \quad \angle 38.15°$

(b) $v = 150 \sin 157t$ [An expression]

(c) $i = \frac{150}{12.71} \sin(157t - 38.15°) = 11.8 \sin(157t - 38.15°)$

(d) $P = \frac{11.8 \times 150}{2} \cos 38.15° - 885 \cos 38.15 \cos 314t$

$+ 885 \sin 38.15° \sin 314t$

$= 695.96 - 695.96 \cos 314t + 545.68 \sin 314t$ watts

$P_{av} = \frac{V_m i_m}{2} \cos\theta = \frac{11.8 \times 150}{2} \cos 38.15° = 695.96$ watts

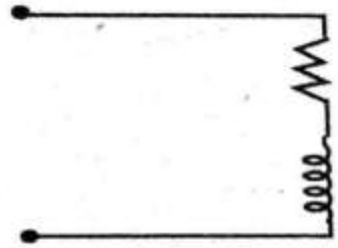
[Remember the average is always equal to dc value = a]

(e) Reactive voltampere $= \frac{v_m i_m}{2} \sin\theta = \frac{150 \times 11.8}{2} \times \sin 38.15$

$= 546.68$ vA

2.29

$x_{RL} = 10 + 7.85 = 12.71 \angle 38.13$



(a) $z_{RL} = \sqrt{R^2 \times x_L{}^2} \angle \tan^{-1} \frac{x_L}{R} = 12.7 \angle 38.2°$

(b) $v = 150 \sin(157t + \theta)$ where, θ = phereshift

$75 = 150 \sin(157t + \theta)$ at $t = 0$

or, $\sin\theta = v_2$

or, $\theta = 30°$

$\therefore$ expression $v = 150 \sin(15yt + 30°)$

(c) As current lags the voltage by 38.2°

$\therefore \quad i = \frac{150}{12.7} \sin(157t + 30° - 38.2°) = 11\text{–}8 \sin(157t - 8.2°)$

(d) $P = \frac{v_m i_m}{2} \cos\theta - \frac{v_m i_m}{2} \cos 2\omega t . \cos\theta = \frac{v_m i_m}{2} \sin 2\omega t \sin\theta$

$= \frac{150 \times 11.8}{2} \cos 38.2 - \frac{150 \times 11.8}{2} \cos(314t - 16.2)$

$\cos 38.2 + \frac{150 \times 11.8}{2} \sin(314t - 16.2) \sin 38.2)$

$= 695.48 - 695.48 \cos(314t - 16.2) + 547.29 \sin(314t - 16.2)$

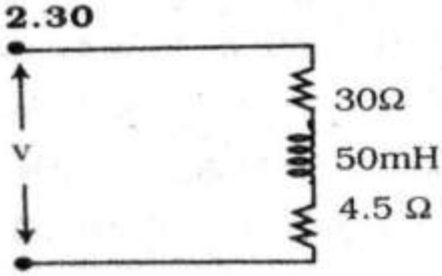
$= 695.48 - 665.8 \cos 314t - 194.03 \sin(314t)$

$+ 525.56 \sin 314t - 152.69 \cos 314t$

$P_{av} = \frac{v_m i_m}{2} \cos\theta = \frac{150 \times 11.8}{2} \times \cos 38 =$

695.48 watts

2.30



Total, $R = 34.5\ \Omega$

$x_L = 18.85$ ohm

$x_{RL} = \sqrt{34.5^2 + 18.85^2} \angle \tan \frac{178.85}{34.5}$

$= 39.31 \angle 28.65$

(a) $i = \frac{100}{39.31} \cos (377t - 28.65)$

$= 2.54 \cos (377t - 28.65)$

(b) $P = v_i = 100 \cos 377t \times 2.54 \cos (377t - 28.65)$

$= 254 \cos 377t (0.87756 \cos 377t + 0.4794v \sin 377t)$

Real power = $114.9 + 114.9 \cos 754t$ watt

Avg. power = 114.9 watts

(d) Reactive voltampere = $60.88 \sin 754t$

Average volue = 0 [∴ sin wave has no area under one cycle]

(e) $x_L = \omega L = 18.89$ ohm

2.31 Given, $i = 10 \cos 157t$ R = 15 ohms and L = 0.0637

(a) $x_l = 0.0637 \times 157 = 10$ ohm

$x_{RL} = \sqrt{15^2 + 10^2} \angle \tan^{-1} \frac{10}{15} = 18.03 \angle 33.69°$

so, $v = 18.03 \times 10 \cos (157t + 33.69)$

$= 180.3 \cos (157t + 33.69)$

(b) $P = vi = \frac{180.3 \times 10}{2} \cos 33.69 - 901.5 \cos 314t . \cos 33.69$

$+ 90.15 \sin 314t \sin 33.69$

$= 750 - 750 \cos 314t + 500 \sin 314t$ watts

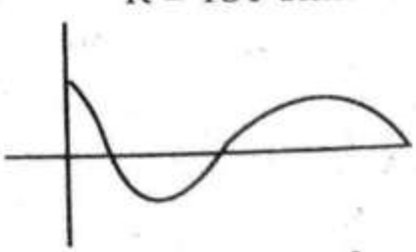
2.32

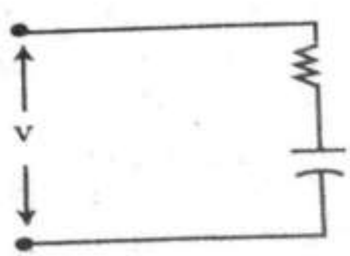
(a) $x_c = \frac{106}{\omega c} = \frac{106}{2\pi \times 60 \times 8} = 331.572$ ohm

(b) $v_c = \frac{106}{2\pi \times 6 \times 106 \times 800} = 33.157$ ohm

2.33

R = 151 ohm





Reactance, $x_c = \frac{1}{\omega c} = (2\pi \times f \times c) = 75.58$ ohm

(a) $v = 15 \cos (2\pi \times 500t) = 15 \cos 3141.6\, t$

(b) $z_{RC} = \sqrt{151^2 + (-x_c)^2} \angle \tan^{-1} \frac{-x_c}{R} = 170.69 \angle -27.79°$

(c) $i = \frac{15}{170.69} \cos (319.6t + 27.79)$

$= 0.0879 \cos (314.6t + 27.79)$

(d) $P = vi = 15 \cos 3141.6t \times 0.0879 \cos (3141.6t + 27.79)$

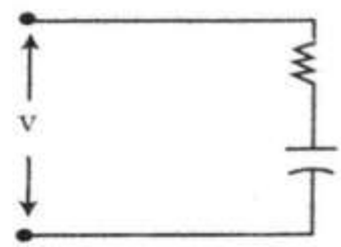
$= 1.3185 \cos 3141.6t\,(0.8847 \cos 3141.6t - 0.466 \sin v\, 3141.6t)$

$= 0.583 + 0.583 \cos 6282\, t - 0.307 \sin 6282t$

2.34 This theory is proved at page 68–69.

2.35

$x_c = \frac{106}{220 \times 303} = 15$ ohm



$v_c = 150 \sin (220t - 60°)$

$i = \frac{150}{15} \sin (220t - 60° + 90°)$

$= 10 \sin (220t + 30°)$ Amp.

How. $z_{RC} = \sqrt{10^2 + (-15)^2} \angle \tan^{-1} \frac{-15}{10} = 18.03 \angle -56.3$

$\therefore$ Voltage drop across the entire, circuit is

$v = 18.03 \times 10 \sin (220t + 30 - 56.3°)$

$= 180.3 \sin (220t - 26.3°)$ volt

2.36 $x_c = \frac{V_m}{I_m} = \frac{20}{0.01} = 2000$ ohm $x_c = \frac{106}{\omega c}$

$c = \frac{106}{\omega\, x_c} = \frac{106}{2\pi \times 2000 \times 2000} = 0.03978\ \mu F$

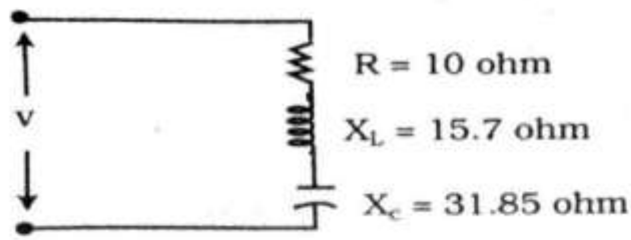
$x_L = \omega L = 157 \times 0.1$

$= 15.7$ ohm

$x_c = \frac{106}{\omega c} = \frac{106}{157 \times 200}$

$= 31.85$ ohm

2.37



a) $i = 10 \sin 157t$,

and voltage v $= iR$

$= 10 \times 10 \sin 157\,t$

$= 100 \sin 157\,t$ volts

(b) $v_l = L.\frac{di}{dt} = 0.1 \times \frac{d}{dt}(10 \sin 157\,t)$

$= 0.1 \times 10 \times 157 \cos 157t$

$= 157 \cos 157t$

(c) $v_c = \frac{q}{c} = \int \frac{i\,dt}{c}$

$= \frac{106}{c} \int 10 \sin 157t\,dt$

$= \frac{106}{200} \times \frac{1}{157} \times 10 - \cos 157t$

$= -318.5 \cos 157t$

From charging law

$q = vc$

and $i = \frac{dv}{dt}$

$q = \int i dt$

(d) Total voltage drop.

$v_{drop} = v_R + v_L + V_C$

$= 100 \sin 157t + 157 \cos 157t + 318.5 \cos 157t$

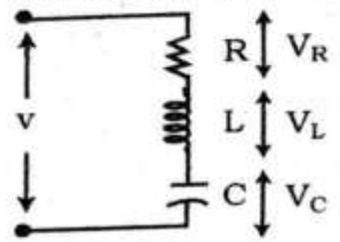
(e) $x_{RLC} = \sqrt{10^2 + (15.7 - 31.85)^2}$

$= 18.99 = 19$ ohm

2.38 This theory is proved at page 74.75.

2.39 $i = I_m \sin (t\sqrt{LC})$

$v_R = I_m R \sin (t\sqrt{LC})$



$$v_L = L\frac{di_L}{dt}$$
$$= L\int I_m \sin (t\sqrt{LC})\, dt$$
$$= L\sqrt{LC}\, I_m\, 8 \cos (t\sqrt{LC})$$
$$= I_m L\sqrt{LC} \cos (t\sqrt{LC})$$
$$v_C = \frac{1}{C}\int I_m \sin (t\sqrt{LC})\, dt$$
$$= -\frac{1}{C}\times I_m \times \frac{1}{\sqrt{LC}} \cos (t\sqrt{LC})$$

$$\therefore \quad W_L = i_L v_L$$
$$= I_m \sin (t\sqrt{LC}) \times I_m \sqrt{LC} \cos (t\sqrt{LC})$$
$$= \frac{I_m^2 \sqrt{LC}}{2} \times L \sin (2t\sqrt{Lc})$$

and $W_C = I_m \sin (t\sqrt{LC}) \times -\frac{1}{C}\times I_m \times \frac{1}{\sqrt{LC}} \cos (t\sqrt{LC})$

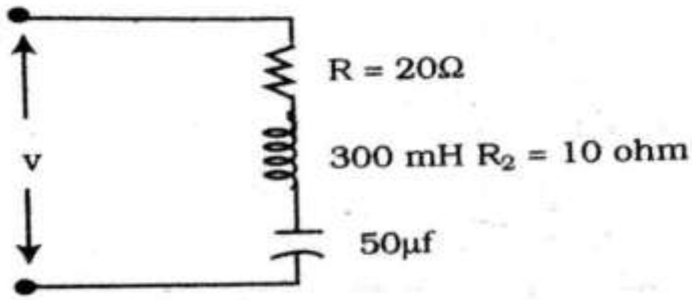
$$= \frac{-I_m^2 \sin (2\sqrt{LCT})}{2C\sqrt{LC}}$$

so, $W_L + W_C = I_m^2\left(\frac{L\sqrt{LC}}{2} - \frac{1}{2C\sqrt{LC}}\right) \sin (2\sqrt{LC}\, t)$

When the circuit resonante at $T = \frac{1}{2\pi\sqrt{LC}}$ or $\omega = \frac{1}{\sqrt{LC}}$

The energy stored by the capacitor will be equal to the energy supplied by the inductor and vice - vessa. Thus oscillation is produced in LC circuit Versa.

2.40



(a) $Z_{RLC} = \sqrt{(20 + 10)^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

$= \sqrt{(30)^2 + \left(157 \times 300 \times 10^{-3} - \frac{1}{30 \times 10^{-6} \times 157}\right)^2}$

$= 85.71\ \Omega$

(b) $i = \frac{100 \angle{-90^\circ}}{20 + 10 + 47 - 1j - 127.388\,j}$

$= 1.167 \angle{-20.488}$

$= 1.167 \cos(157t - 20.488)$

(c) $P = vi$

$= 100 \angle{-90^\circ} \times 1.167 \angle{-20.488}$

$= 116.67 \angle{-110.488}$

(d) Avg power = −40.83

(e) Do yourself having experienced with previous similar mathematics.

(f) $v_R = 23.34$

2.41 $v = 282.8 \sin 500t$

and $i = 5.656 \sin(\text{toot} - 36.870)$

In phasor algebra,

$v = 282.8 \angle 0^\circ$

and $i = 5.656 \angle{-36.87^\circ}$

$Z = \frac{282.8}{5.656 \angle{-36.87^\circ}}$

$= 50 \angle 36.87$

Let it is a RLC circuit,

$\sqrt{(x_L.x_C)^2 + R^2} \angle \tan^{-1} \frac{(-x_C + x_L)}{R}$

$= 50 \angle 36.8$

$(x_L - x_C)^2 + R^2 = 50 - 11)$

and $\frac{x_L - x_C}{R} = \tan 36.87$

where, $x_C = \frac{1}{\omega c} = \frac{1}{500 \times 100\ \mu}$

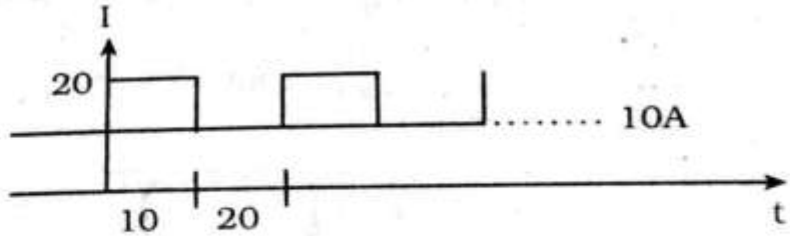
$= 20\ \Omega$

Chapter–3

Effective Current and Voltage Average Power

3.1 $i(10) = 20$

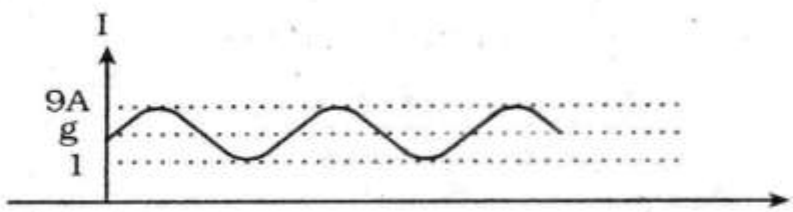
$i(20) = 10$ amp



$$i_{eff} = \sqrt{\frac{20^2 \times 10 + 10^2 \times 20}{30}}$$

$= 14.14$ amp

3.2 **(a)**



(b) $i(t) = 5 + 4 \sin \omega t$

(c) $i_{avg} = 5$ amp

(d) $$i_{eff} = \sqrt{\frac{1}{\frac{T}{2}}\int_{0}^{\frac{T}{2}} (5 + 4 \sin \omega t)^2 \, dt}$$

$$i_{eff} = \sqrt{\frac{1}{\frac{T}{2}}\int_{0}^{\frac{T}{2}} (5 + 4 \sin \omega t)^2 \, dt}$$

$$= \sqrt{\frac{2}{T}\int_{0}^{\frac{T}{2}} (25 + 40 \sin \omega t + 16 \sin^2 \omega t) \, dt}$$

$$= \sqrt{\frac{2}{T}\left[25 \times \frac{T}{2} + \frac{40}{\omega}\cos\frac{\omega T}{2} - \frac{40}{\omega} + \frac{1}{2} \times 16 - \frac{16}{2 \times 2\omega}\sin 2\omega\frac{T}{2}\right]}$$

$$= \sqrt{\frac{2}{T}\left[\frac{25T}{2} + \frac{40}{\omega}(-1) - \frac{40}{\omega} + 8 - 0\right]}$$

$= 5.75$ amp

3.3 $e = 50t$

form factor $= \frac{I_{rms}}{I_{avg}}$

$$I^2_{rms} = \frac{1}{T}\int_0^T (50t)^2\, dt$$

$$= \frac{1}{3}\int_0^3 50^2 \times t^2 dt$$

$$= \frac{1}{3} \times 50^2 \times \frac{t^3}{3}\Big|_0^3$$

$$= \frac{1}{3} \times 50^2 \times \left[\frac{3^3}{3} - 0\right]$$

$$= \frac{1}{3} \times 50^2 \times 3^2$$

$= 7500$

$I_{rms} = 86.60$

$\therefore$ form factor $= \frac{86.60}{75} = 1.155$

$$I_{avg} = \frac{1}{T}\int_0^T 50t\, dt$$

$$= \frac{50}{3}\frac{t^2}{2}\Big|_0^3$$

$$= \frac{50}{3} \times \frac{3^2}{2}$$

$$= \frac{50}{2} \times 3$$

$= 25 \times 3 = 75$

3.4 From the previous math $i_{eff} = 86.6$ amp

$\therefore$ crest factor $= \frac{50}{86.6} = 0.577$

3.5 $i_1 = 5 \sin \omega t$

$i_2 = 10 \sin(\omega t + 60^\circ)$

$i_2 - i_1 = 10 \sin(\omega t + 60^\circ)\ 5 \sin \omega t$

$= 8.66 \angle 90^\circ$

$= 8.66 \sin(\omega t - 90^\circ)$

3.6 R = 5 omh

$L = \frac{10}{377}$

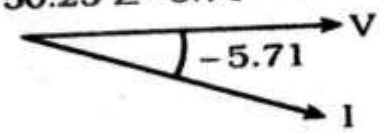
$X_L = \omega L = 10\Omega$

$V = V\angle 0^\circ$

$I = \frac{V\angle 0^\circ}{50 + (10 - 16)j}$

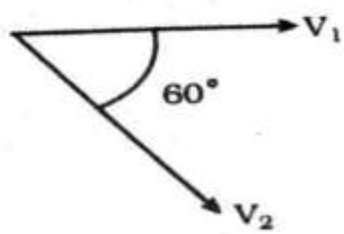
$X_C = 15\Omega$ $\quad = \dfrac{V \angle 0^\circ}{50 - 5j}$

$= \dfrac{V \angle 0^\circ}{50.25 \angle -5.71}$



3.7 $V_1 = 100 \angle 0^\circ$

$V_2 = 100 \angle -60^\circ$



(a) So $V = V_1 + V_2$

$= 100 \angle 0^\circ + 100 \angle -60^\circ$

$= 173.2 \angle 30^\circ$

(b) $V = V_1 - V_2$

$= 100 - 100 \angle -60^\circ$

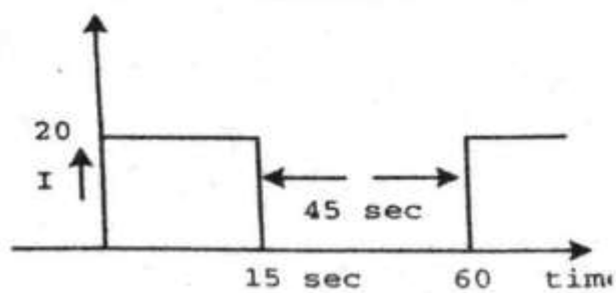
$= 100 \angle 60^\circ$.

3.8 Average current,

$$I = \sqrt{\frac{1}{60}(20 \times 15)}$$

$= 10$ amp

[Tips : Average $x_{signal} = \dfrac{\text{Area (integration)}}{\text{total time}}$]



Total time period

$T = 15 + 45 = 60$ sec

Let, the resistance of the elevator be R.

so, $P = I^2R = \frac{W}{T} = \frac{i^2Rt}{T}$

$\therefore \quad I^2 = \frac{i^2t}{T} = \frac{20^2 \times 15}{60} = 100$ amp or $I = 10$ amp

$\therefore$ Total work done in one cycle

$W = I^2Rt = 20^2.R.15$

$= 6000\ R$

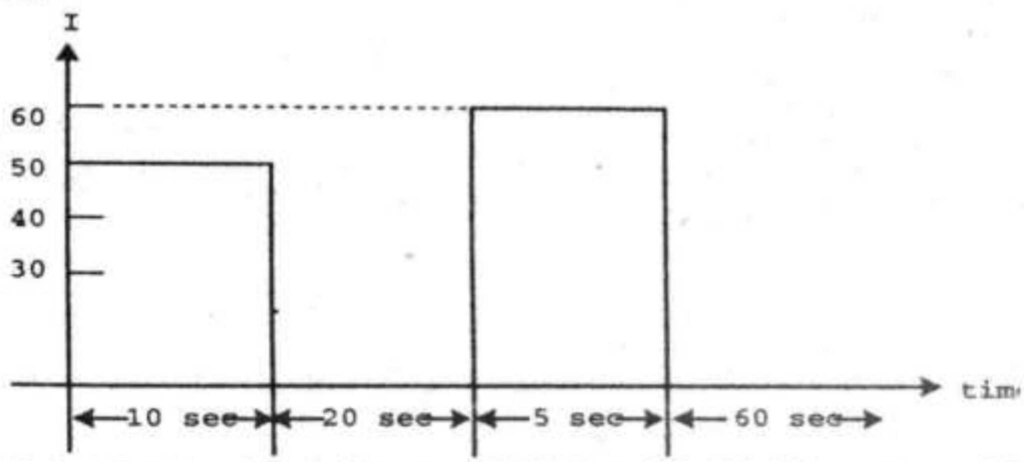
Now, $P = \frac{W}{t} = \frac{6000\ R}{60} = 100R$

Let, the equivalant continous current be I_1

$P = I_1{}^2R = 100R$

or, $I_1 = 10$ amp

3.9



Substituting the value x_L = (20R tan 36.87) 50 equation (i)

$(20 + R\tan 36.87 - 20)^2 + R^2 = 50$

or, $(0.75)^2 R^2 + R^2 = 50$

or, $(0.5625 + 1) R^2 = 50$

or, $R^2 = \frac{50}{1.5625}$

or, $R^2 = 32$

or, $R = 5.65\ \Omega$

$\therefore$ Resistance $R = 5.65\ \Omega$

Now, substituting the value at

$X_L = 20 + R \tan 36.87$

$= 20 + 4.243$

$= 24.243\ \Omega$

so, $L = 0.0485$ H

$= 48.5$ mH

So, R = 5.65 Ω, L = 48.5 mH are the rest of the circuit parameter.

Power needed until the motor gets overheated,

$P = \sqrt{50^2 \times 10 + 60^2 \times 5}$

$= 207.36$ watt

3.10 The period,

$T = 0.3 + 0.5 + 0.4 + 0.5 + 0.3 = 2$ sec

(a) Average current,

$$I_{av} = \frac{10\times0.3 + 0.5 \times 15 + 50 \times 0.4 + 15 \times 0.54 + 10\times0.3}{2}$$

$= 20.5$ amp

(b) $$I = \sqrt{\frac{10^2\times0.3 + 15^2\times0.5 + 50^2\times0.4 + 10^2\times0.3 + 15^2 \times 0.5}{2}}$$

$= 25.35$ amp

3.11

(a) d.c ammeter shows the average current. So, the reading will be 20.5 A

a - c ammeter shown the effective current. So, the reading will be 25.35A

(b) a - c reading should be employed.

3.12 Energy produced in one cycle is

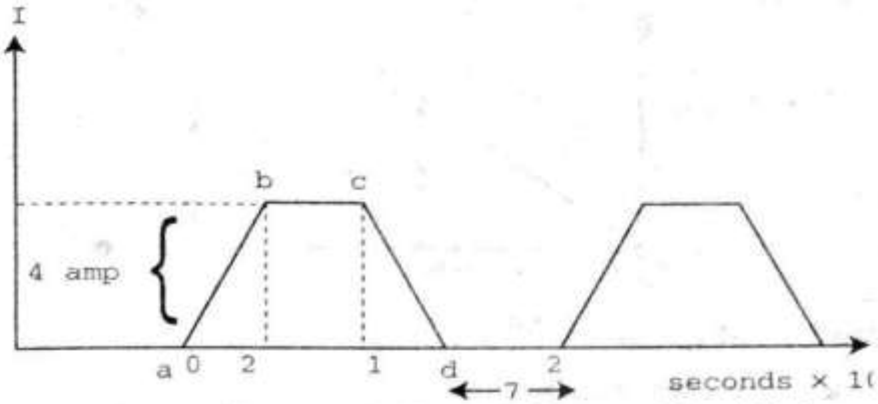
(a) $I^2 Rt = 25.35^2 \times 5 \times 2 = 6426.23$ Joules

$= \frac{6426.23}{4.184} = 1535.9$ cal

[1 cal = 4.2 Joules]

(b) $P = I^2R = 25.39^2 \times 5 = 3213.1$ watt

3.13



Time period $T = (2 + 1 + 217) \times 10^{-4}$ sec $= 12 \times 10^{-4}$ sec

(a) Frequency $n = \frac{1}{T} = \frac{1}{12 \times 10^{-4}} = 833.33$ Hz

Eq. of the curve as is,

$$i = \frac{4}{2 \times 10^{-4}} t$$

or, $i = 2 \times 10^{4} t$

From figure ωe sec that curve cd is similar to ab.

$$\text{(b)} \quad I_{av} = \frac{\int_{2}^{2\times10^{-4}} 2\times10^{4}\, tdt + 4\times1\times10^{-4} + \int_{0}^{2\times10^{-4}} 2\times10^{-4}\, tdt}{12\times10^{-4}}$$

$$= \frac{2 \times 2 \times 10^{4} \int_{0}^{2\times10^{-4}} t.dt + 4 \times 10^{-4}}{12 \times 10^{-4}}$$

$$= \frac{4 \times 10^{4} \times \frac{1}{2} (2 \times 10^{-4})^2 + 4 \times 10^{-4}}{12 \times 10^{-4}}$$

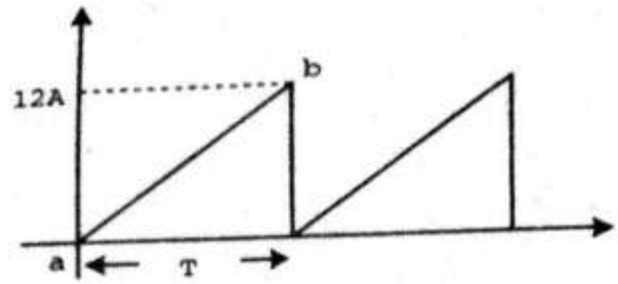
$$= \frac{12 \times 10^{-4}}{12 \times 10^{-4}} = 1 \text{ amp}$$

$$\text{(c)} \quad I = \sqrt{\frac{2 \times \int_{0}^{2\times10^{-4}} (2 \times 10^{4} t)^2 dt + 4^2 \times 10^{-4}}{12 \times 10^{-4}}}$$

$$= \sqrt{\frac{2 \times 4 \times 10^{8} \times \left. \frac{t^3}{3} \right|_{0}^{2\times10^{-4}} + 4^2 \times 10^{-4}}{12 \times 10^{-4}}}$$

$$= \frac{\sqrt{\left(\frac{64}{3} + 16\right) \times 10^{-4}}}{12 \times 10^{-4}} = 1.76 \text{ amp}$$

3.14



Equation of ab in $i = \frac{12}{T} t$

$\therefore$ a – c meter reading.

$$I = \sqrt{\frac{1}{T}\int_0^T \left(\frac{12}{T} t\right)^2 dt}$$

$$= \sqrt{\frac{1}{T} \times \frac{144}{T^2} \times \left[\frac{t^3}{3}\right]_0^T}$$

$$= \sqrt{\frac{1}{T} \times \frac{144}{T^2} \times \frac{T^3}{3}}$$

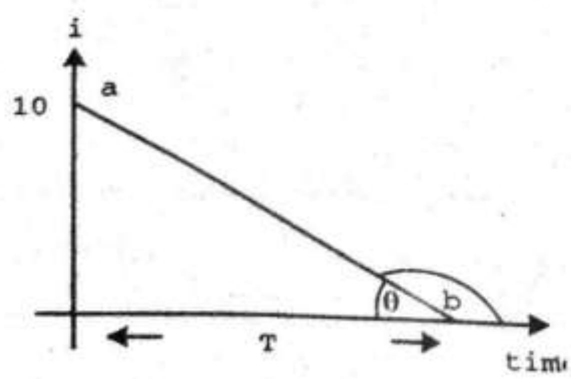
$$= \frac{12}{\sqrt{3}} = 6.93 \text{ A}$$

3.15 Slop = tan $(\pi - \theta) = \tan\theta$ equation of line ab

$$\frac{i - 0}{0 - 10} = \frac{t - T}{T - 0}$$

or. $\frac{i}{-10} = \frac{t - T}{T}$

or. $i = \frac{10t}{T} + 10$



$\therefore \quad i = \frac{-10}{T} t = 10$

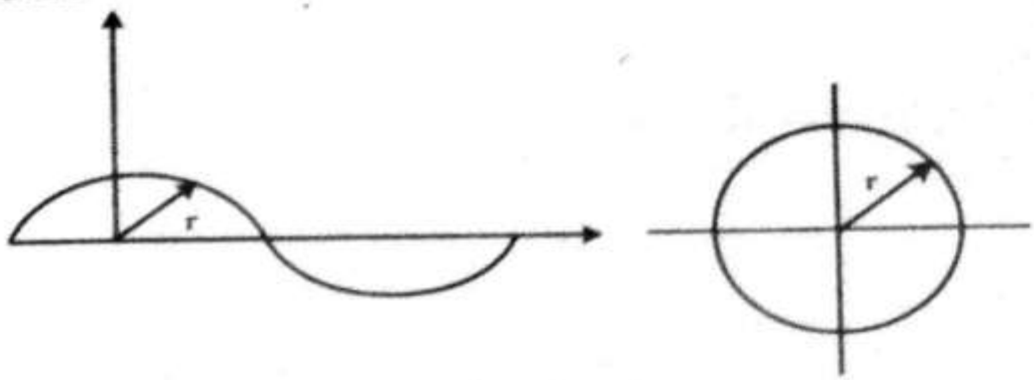
$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T \left(\frac{100t^2}{T^2} + 2\left(\frac{-10t}{T}\right)10 + 100\right)dt}$$

$$= \sqrt{\frac{1}{T}\left(\frac{100T^3}{T^2 \times 3} + 2\left(\frac{-10\,T^2}{2T}\right)10 + 100T\right)dt}$$

$$= \sqrt{\frac{1}{T}\left(\frac{100T}{3} - 100T + 100T\right)}$$

$$= \sqrt{\frac{100}{3}} = 5.77\text{ A}$$

3.16



As sine function is a circular function.

The equation of the circle is

$$t^2 + i^2 = P^2$$

$$i^2 = (P^2 - t^2)$$

$$= \sqrt{\frac{P^2 - t^2}{2P}dt}$$

$$\therefore \quad I_{rms} = \sqrt{\frac{\int_{P}^{P} i^2\,dt}{\int_{-P}^{P} dt}}$$

$$= \sqrt{\frac{1}{2P}\left[P^2t - \frac{t^3}{3}\right]_{-P}^{P}}$$

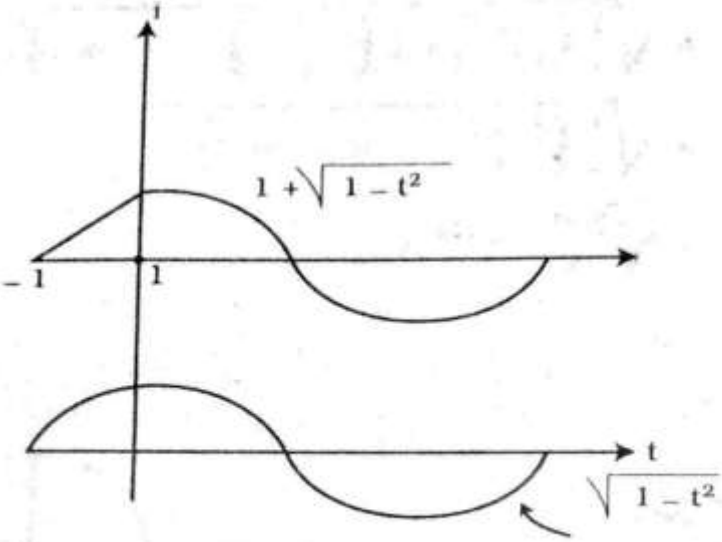
$$= \sqrt{\frac{1}{2P}\;P^2(P + P) - \frac{P^3 + P^3}{3}}$$

$$= \sqrt{\frac{4}{6}P^2}$$

$$= 0.816\text{ P·amp}$$

3.17

$\therefore$ E_q^n of the resultant current $= 1 + \sqrt{1 - t^2}$



$$\therefore \quad I_{rms} = \int_1^1 \frac{1 + \sqrt{1 - t^2}}{1 + 1}$$

$$= \sqrt{\frac{1}{2}\left(1 + 2\sqrt{1 - t+2} + 1 - t^2\right) dt}$$

$$= \sqrt{\int_{-1}^{1} \frac{(2 + 2\sqrt{1 - t^2} - t^2)dt}{2}}$$

As we know, $\int \sqrt{1 - t^2}\, dt = \frac{t\sqrt{1 - t^2}}{2} + \frac{1}{2}\sin^{-1} t$

so, $\int_{-1}^{1} \sqrt{1 - t^2}\, dt = \left[\frac{t\sqrt{1 - t^2}}{2} + \frac{1}{2}\sin^{-1} t\right]_{-1}^{1}$

$$= \frac{1}{2}\sin^{-1} 1 - \frac{1}{2}\sin^{-1}(-1) = \frac{1}{2} \times 180^\circ = 90^\circ$$

or, $\int_{-1}^{1} \sqrt{1 - t^2}\, dt = \frac{\pi}{2}$

$\int_{-1}^{1} 2dt = 2t\,\Big|_{-1}^{1} = 4$

and $\int_{-1}^{1} t^2 = \frac{t^3}{3} = 0$ [odd function] so, $I_{pms} = \sqrt{\int_{-1}^{1} \frac{(2 + 2\sqrt{1-t^2} - t^2)}{2} dt}$

$= \sqrt{\frac{4 + \frac{2\pi}{2}}{2}}$

$= \sqrt{2 + \frac{\pi}{2}}$

$= 1.89$ amp

3.18 Form factor $= \frac{I_{rms}}{I_{avg}}$

$= \frac{6.93 \text{ A}}{I_{avg}}$ [From ans of 14]

Here,

$I_{avg} = \frac{1}{T}\int_0^T \frac{12}{T} t \, dt$

$= \frac{1}{T} \times \frac{12}{T} [t^2]_0^T$

$= \frac{12}{T^2} \times T^2$

$= 12$

So, form factor $= \frac{6.93}{12} = 0.5775$

Here, 19, 20, 21, 22 are same in mathematical procedure.

I am giving an example of 21 for help to find the procedure.

3.19

$$e_{rms} = \sqrt{\frac{1}{T}\int_0^T (100 \sin \omega t + 60 \sin(5\omega t + 30°))^2 dt}$$

$$= \sqrt{\frac{1}{T}\int_0^T [100^2\sin^2\omega t + 2 \times 100 \times 60 \sin \omega t \sin(5\omega t + 30°) + 60^2\sin^2(5\omega t + 30)]dt}$$

$$= \sqrt{\frac{1}{T}\int_0^T 100^2\sin^2\omega t dt + \int_0^T 2 \times 100 \times 60 \sin \omega t \sin(5\omega t + 30°) + \int_0^T 60^2\sin^2(5\omega t + 30) \, dt}$$

it is intoitive that the integration can be done easily from your previous

Mathematical experience

3.20

$$e_{rms} = \sqrt{\frac{1}{T}\int_0^T [100^2 \sin^2 \omega t + 2 \times 100 \times 60 \sin \omega t \cos 3\omega t + 60^2 \cos^2 3\omega t]dt}$$

$$= \sqrt{\frac{1}{T}\left[\int_0^T 100^2 \sin^2 \omega t dt + \int_0^T 2 \times 100 \times 60 \sin \omega t \cos 3 \omega t dt + \int_0^T 60^2 \cos^2 3 \omega t dt\right]}$$

It is intuitive that the integration can be done easily from your past mathematical experience.

$$e_{avg} = \sqrt{\frac{2}{T}\left[\int_0^{\frac{T}{2}} 100 \sin \omega t + \int_0^{\frac{T}{2}} 60 \cos 3\omega t\right]}$$

$$= \sqrt{\frac{2}{T}\left[\frac{100}{\omega}\cos\frac{\omega T}{2} - \frac{100}{\omega} + \frac{60}{\omega}\sin\frac{3\omega T}{2} - 0\right]}$$

$$= \sqrt{\frac{2}{T}\left[+\frac{100}{\omega} + \frac{100}{\omega}\right]}$$

$$= \sqrt{\frac{2}{2\pi}[+200]}$$

$$= \sqrt{\frac{200}{\pi}} = 7.978 \text{ amp}$$

3.21

$$e_{rms} = \sqrt{\frac{1}{T}\left[\int_0^T 100^2 \sin^2 \omega t dt - \int_0^T 40 \times 100 \times 2 \sin \omega t \sin 3\omega t \, dt + \int_0^T 40^2 \sin^2 3\omega t dt\right]}$$

It is intuitive that the integration can be done easily from your previous experience.

3.22 $$e_{avg} = \sqrt{\frac{1}{\frac{T}{2}}\left[\int_0^{\frac{T}{2}} 100 \sin \omega t - \int_0^{\frac{T}{2}} 40 \sin 3\omega t dt\right]}$$

It is intuitive that the integration can be done easily from your past exerience.

Hence form factor $= \frac{e_{rms}}{e_{avg}}$

3.23

$$e_{rms} = \sqrt{\frac{1}{T}\int_0^T e^2}$$

$$= \sqrt{\frac{2}{T}\int_0^{T/2} (100 \sin \omega t - 40 \sin 3\omega t)^2 dt}$$

$$= \sqrt{\frac{2}{T}[\int_0^{T/2} (100)^2 \sin^2\omega t - 2\times 100\times 40 \sin \omega t \sin 3\omega t + (40)^2 (1 - \cos 6\omega t] dt}$$

$$= \sqrt{\frac{2}{T}\int_0^{T/2} [10^4 (1 - \cos 2\omega t) - 2\times 100\times 40 \{\cos 4\omega t - \cos 2\omega t\} + (40)^2 (1 - \cos 6\omega t)]}$$

$$= \sqrt{\frac{2}{T}[10^4 \int_0^{T/2} (1 - \cos 2\omega t)dt - \int_0^{T/2} 2\times 100\times 40 \{\cos^4\omega t - \cos 2\omega t\}dt + \int_0^{T/2} (40)^2 (1 - \cos 6\omega t)dt]}$$

. The answer is intuitive.

3.24 Rectangular wave

3.25 Square wave's form factor.

3.26 Peak factor $= \dfrac{\text{max}^{m}\text{ value}}{\text{rms value}}$

(a) sine wave.

$$I_{rms} = \sqrt{\frac{2}{T}\int_0^{T/2} \sin^2\omega t \, dt}$$

$$= \sqrt{\frac{2}{T}\int_0^{T/2} \frac{1}{2}(1 - \cos 2\omega t)dt}$$

$$= \sqrt{\frac{2}{2T}\left(\frac{T}{2} - \frac{\sin 2\omega t}{2\omega}\Big|_0^{T/2}\right)}$$

$$= \sqrt{\frac{2}{2T}\left(\frac{T}{2} - \frac{\sin 2\omega \frac{T}{2}}{2\omega}\right)}$$

$$= \sqrt{\frac{2}{2T}\left(\frac{T}{2}\right)}$$

$$= \sqrt{\frac{1}{2}} = 0.707$$

$\therefore$ Peak factor $= \sqrt{2}$

(b) rectangular wave.

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T a^2\, dt}$$

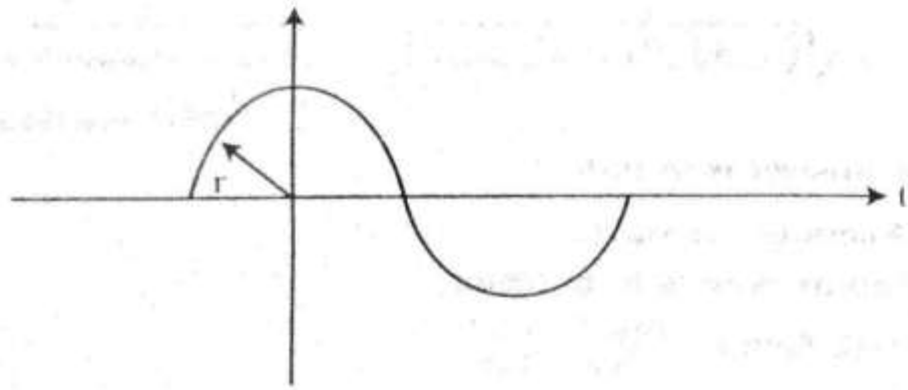
$$= \sqrt{\frac{1}{T} \times a^2 T}$$

$$= a$$

$\therefore$ Peak factor $= \frac{a}{a} = 1$

3.27

Creast factor $= \frac{\text{max}^m \text{ value}}{\text{rms value}} = \frac{50}{25.35} = 1.972$ amp



let, the redius be r

From promblem (16) we have $I_{rms} = 0.816\ r$

$$I_{avg.} = \frac{1}{2r}\int_{r}^{r} \sqrt{r^2 - t^2}\, dt$$

$$= \frac{1}{2r}\left[\frac{t\sqrt{r^2 - t^2}}{2} + \frac{r^2}{2}\sin^{-1}\frac{t}{r}\right]_r^r$$

$$= \frac{1}{2r}\left[0 + \frac{r^2}{2}\sin^{-1} 1 - 0 - \frac{r^2}{2}\sin^{-1}(-1)\right]$$

$$= \frac{1}{2r}.r^2.\frac{\pi}{2} = \frac{\pi r}{4} = 0.785\ r$$

$\therefore$ Creast factor $= \frac{I_{max}}{I_{rms}} = \frac{r}{0.816r} = 1.225$

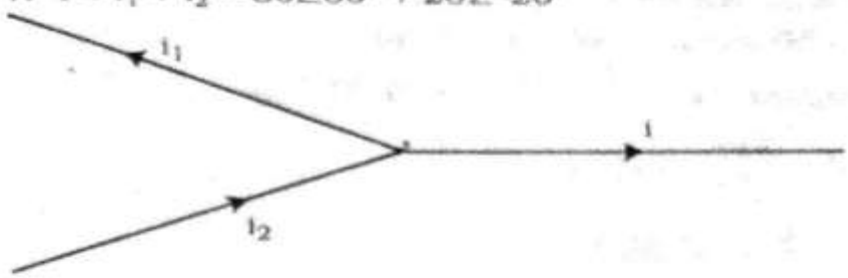
and from factor $= \frac{I_{rms}}{I_{av}} \frac{0.816\ r}{0.786\ r} = 1.04$

3.28

$i_1 = 30 \sin(\omega t + 60°) = 30\angle 60°$

$i_2 = 20 \sin(\omega t - 20) = 20\angle -20°$

$\therefore\ i = i_1 + i_2 = 30\angle 60° + 20\angle -20°$



$\therefore\quad i = 15 + i\,25.98 + 18.79 - i\,6.84 = 38.83\ \angle 29.53$

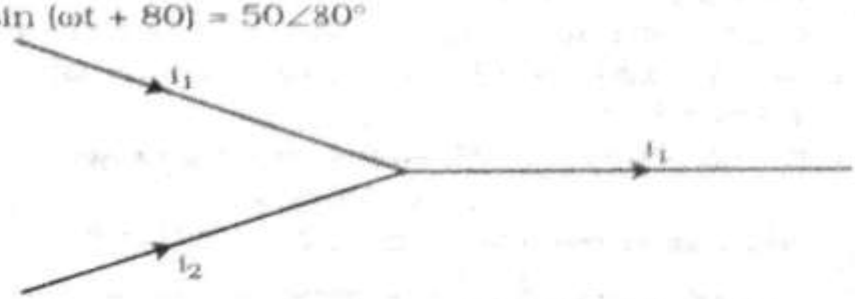
$\therefore\quad i = 38.83 \sin(\omega t + 25.53)$

$\therefore\quad$ Effective current $= \dfrac{38.83}{\sqrt{2}} = 27.65$ A

3.29

$i = 40 \sin(\omega t - 40°) = 40\angle -40°$

$i = 50 \sin(\omega t + 80) = 50\angle 80°$



$i \quad = i_1 + i_2 \quad$ or. $i_2 = i - i_2 = 50\angle 80 - 40\ \angle -40$

or. $i_2 \quad = 8.68 + i\,49.24 - 30.64 + i\,25.71$

$\quad = 70\ \angle 106.05°$

$\therefore\quad i_2\ \ 70 \sin(\omega t + 106.05)$ A

3.30 $I = 25$A. $V = 220$ v. $\cos\theta = 0.88$

$\therefore\quad$ Power $= VI\cos\theta = 25 \times 220 \times 0.88 = 4040$ watt

VARS $VI \sin\theta = 25 \times 220 \times \sqrt{1 - (0.88)^2} = 2612.36 \times V_{ars}$

Reactive factor. $\sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - (0.88)^2} = 0.475$

3.31 $I = 10$A. $V = 200$ v. $\cos\theta = 0.8$

$\therefore\quad$ Power $= VI\cos\theta = 10 \times 220 \times 0.8 = 1760$ watts

$\therefore\quad$ Reactive volt – ampere $= VI\sin\theta = 10 \times 220 \times \sqrt{1 - (0.8)^2}$

$= 1320$ VARS

Reactive factor, $\sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - (0.8)^2} = 0.6$

volt – ampere = VI = 10 × 220 = 2200 VA

3.32

$V = 20 \sin(\omega t + 30) = 200\angle 30$

$i = 50 \sin(\omega t + 60^\circ) = 50\angle 60^\circ$

$\therefore$ Impedence $= \dfrac{200\angle 30}{50\angle 60} = 4\angle 0.30$

Now, $V = \dfrac{200}{\sqrt{2}} = 141.\ 42$ v

$I = \dfrac{50}{\sqrt{2}} = 35.36$ A

Average power, $P = VI\cos\theta = 141.42 \times 35.36 \times \cos(-30^\circ)$

$= 4330.6\ \omega$

volt – ampere = VI = 141.42 × 35.36 = 5000 v_{ar}

Power factor = cos (–30°) = 0.866

3.33

I = 15A, V = 220 v, θ = 72°

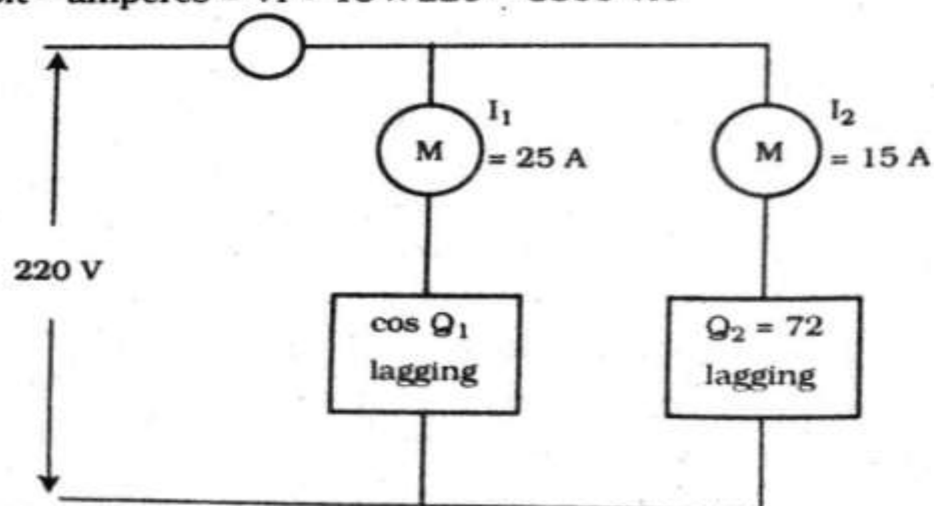
(when current is lagging, the cosθ is lagging power P = VI cosθ = 15 × 220 cos 72° factor and vice - versa)

= 1019.75 r

Reactive power = VI sinθ = 15 × 220 × sin 72

= 3138.49 vars

volt – amperes = VI = 15 × 220 = 3300 VA



$\cos\theta_1 = 0.88$ and $\theta_1 = 28.36^\circ$

Since the p.f in lagging power factor.

$\therefore \quad I_1 = 25\angle{-28.36}$

$I_2 = 15\angle{-72°}$

$\therefore \quad I = I_1 + I_2 = 25\angle{-28.36} + 15\angle{-72}$

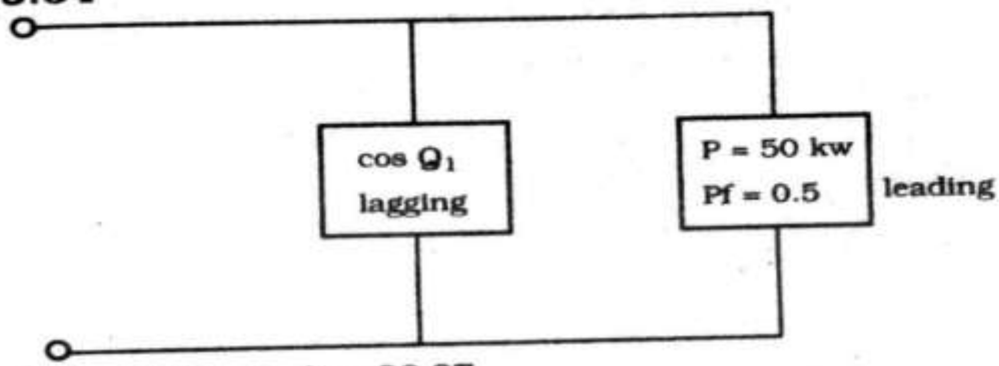
$= 37.32 \angle{-44.46°}$

$\therefore$ volt – ampere = VI = 220 × 37.32 = 8210.4 VA

$\therefore$ total current I = 37.32 A

Power, P = VI cosθ = 8210.4 cos 44.46° = 5860.09 watt

3.34



$\cos\theta_1 = 0.8 \quad \therefore \theta_1 = 36.37$

$\cos\theta_1 = 0.9 \quad \therefore \theta_2 = 80°$

Since power factor is lagging, $I_1 = 250\angle{-36.87°}$

Now, $VI_2 \cos\theta_2 = 50 \times 10^3$ or, $220 \times I_2 \times 0.05 = 50{\times}10^3$

$\therefore \quad I_2 = 454.55A$

Sine leading power factor.

$\therefore \quad I_2 = 454.55\angle{60°}$

$I = I_1 + I_2 = 250\angle{-35.87°} + 454.55\angle{60°}$

$= 491.86 \angle{29.6937}$

$\therefore$ line current = 491.86 amp

Power current is leading, p.f is leading.

3.35

$v = 200 \sin \omega t$

$\therefore \quad v = \frac{200}{\sqrt{2}} \angle 0$

[RMS = Peak/$\sqrt{2}$]

$i = 50 \cos (\omega t - 30°$

$= 50 \sin (\omega t - 30° + 90°)$

$= 50 \sin (\omega t + 60°)$

$I = \frac{50}{\sqrt{2}} \angle 60° = 3535 \angle 60°$

$\therefore \quad Z = \frac{141.421\angle 0°}{35.355\angle 60°} = 42\angle 60°$

$\therefore$ Average power = VI $\cos\theta$ = 141.421 × 35.355 cos60°
= 2500 WaHs

Vars = VI $\sin\theta$ = 4330 vars
Power factor = cos 60° = 0.5

3.36

VI $\sin\theta$ = 600 ... ... (i)
VI $\cos\theta$ = 800 ... ... (i)

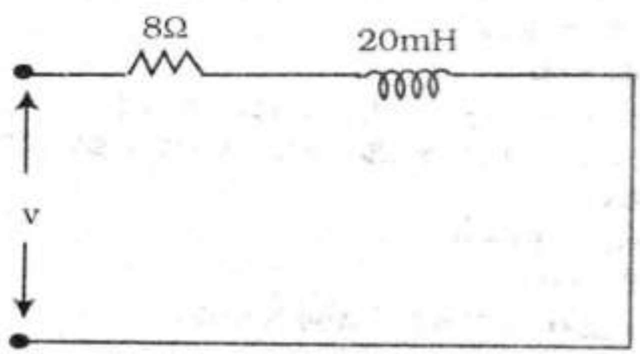
Now,

$$VA = \sqrt{(real)^2 + (imaginary)^2} = \sqrt{(VI\cos\theta)^2 + (VI\sin\theta)^2}$$
$$= \sqrt{600^2 + 800^2} = 1000 \text{ VA}$$

Power factor $\cos\theta$ = cos 36.87° = 0.8
Reactive factor $\sin\theta$ = sin 36.87° = 0.6

3.37

$X_L = \omega L$
$= 2\pi \times 60 \times 20$ mH
= 7.54 ohm



$Z = 8 + 7.54 = 11\angle 13.3°$ current = $\frac{110}{11}$ = 10 amp

Power, P = $VI\cos\theta$ = 110 × 10 × cos 43.3 = 800.5 A

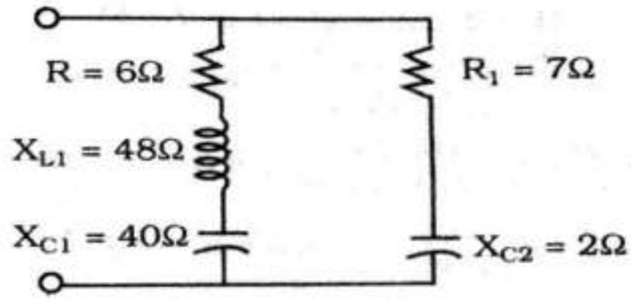
3.38

$Z_1 = 6 + j(48 - 40) = 10\angle 53.13$
$Z_2 = 7 - j2 = 7.28\angle -15.95$

$$\therefore Z_{circuit} = \frac{Z_1 \times Z_2}{Z_1 + Z_2}$$
$$= \frac{10\angle 53.13 \times 7.28\angle -15.95}{6 + j8 + 7 - 2j}$$

$= \frac{72.8\angle 37.18}{14.32\angle 24.78}$

$= 5.08\angle 12.4$



Current $= \frac{100}{5.08} = 19.88$ amp

Total power = VI cosθ = 100 × 19.68 = cos 12.4 = 1922 ω

$I_1 = \frac{V}{Z_1} = \frac{100}{10} = 10A$

∴ $P_1 = VI_1 \cos\theta_1 = 100 \times 10 \cos 53.13 = 600$ ω

$I_2 = \frac{V}{Z_2} = \frac{100}{7.28} = 13.74$

∴ $P_2 = 100 \times 13.74 \cos(-15.95) = 1321.1$ WaH.

3.39

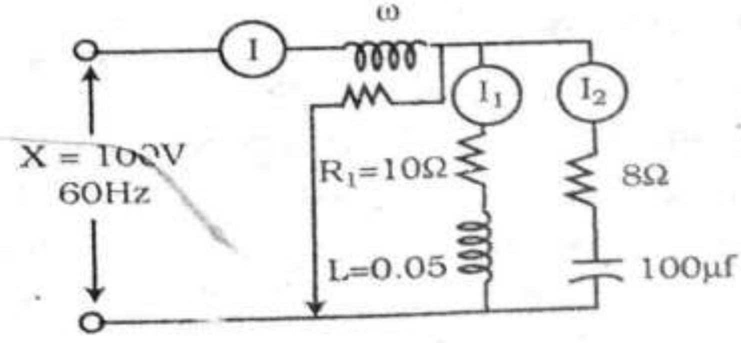
$X_{L1} = \omega L = 2 \times 60 \times 0.05$

$= 18.85$ ohm

$X_{02} = \frac{106}{377 \times 100} = 26.53$0hm

$Z_1 = 10 + j\,18.85 = 21.338\angle 62$

$Z_2 = 8 - j26.53 = 27.71\ \angle -73.22$



∴ Impedence of the circuit is–

$$Z_0 = \frac{Z_1 \times Z_2}{Z_1 + Z_2}$$

$$= \frac{21.388\angle 62 \times 27.71\ \angle{-73.22}}{21.338\angle 62 + 27.71\ \angle{-73.22}}$$

$$= \frac{591.276\angle{-11.22}}{19.57\angle{-23.1}} = 30.213\angle 11.88$$

$$\therefore \quad I = \frac{V}{Z_0} = \frac{100\angle 0}{30.213\angle 11.86} = 3.31\angle{-11.88^\circ}$$

$$I_1 = \frac{V}{Z_1} = \frac{100\angle 0}{21.338\angle 62} = 4.606\angle{-62}$$

$$I_2 = \frac{V}{Z_2} = \frac{100\angle 0}{27.71\angle{-73.22}} = 3.61\angle 13.22$$

$P = VI\cos\theta = 100 \times 3.31 \times \cos 11.88 = 323.91$ WaH

∴ The ammeter readings will be $I_1 = 4.686$ A

$I_2 = 3.61$ A

$I = 3.31$ A

∴ The watt meter readings will be $P = 323.91\ \omega$

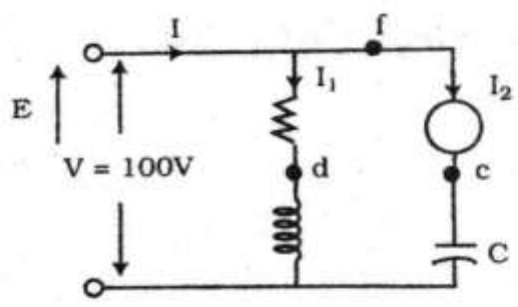


Fig. (a)

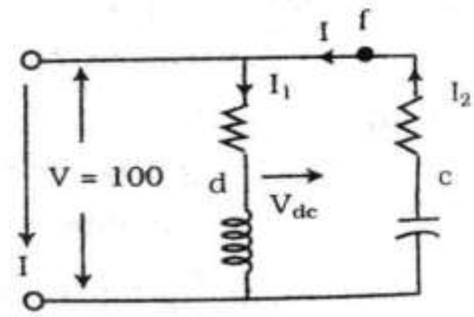


Fig. (b)

(c) According to kirchoffis voltage law

$$-V_{dc} + V_{cf} + V_{fd} = 0$$

$$V_d = V_{cf} - V_{fd} = V_{fc} - V_{fd} = I_1R_1 - I_1R_2$$

$$= 4.686\angle{-62} \times 10 - 3.61\angle 73.22 \times 8$$

$$= 22 - 41.375 - 8.338 - 27.65$$

$$= 70.364\angle{-78.8}\ \text{V}$$

(d) for fig (b)

According to kirchoff's voltage law–

$V_{dc} + V_{cf} + V_{fd} = 0$

or. $V_{dc} = V_{cf} - V_{fd} = V_{df} - V_{cf}$

$= 3.61\angle 73.32 \times 8 - 4.686\angle -62 \times 10$

$= 8.338 + 27.65 - 22 + 41.375$

$= 70.364\angle 101.2°$ volt

3.40 $R_1 = 8$ omh

$L_1 = 0.025$

$X_{L1} = \omega l = 2\pi \times 0.025 \times 60 = 0.157 \times 60 = 9.42$

$X_{C1} = \frac{1}{\omega C} = \frac{1}{2\pi \times 120\pi f \times 60}$

$= \frac{1}{2\pi \times 120\ \mu f \times 60}$

$= 22.10\Omega$

$I_1 = \frac{100}{8 + 9.42j}$

$= 8.09 \angle -49.66$ A

$I_2 = \frac{100}{8 - 22.10j}$

$= 4.254 \angle 70.100$ A

(b) Do yourself

(c) $V_{dc} = 10 \times 8.09 \angle -49.66 - 8 \times 4.254 \angle 70.1$

$= 102.157 \angle -66.469$

3.41 (a) $I = I_1 + I_2 = 7.027$

(b) $\omega = VI \cos \theta$

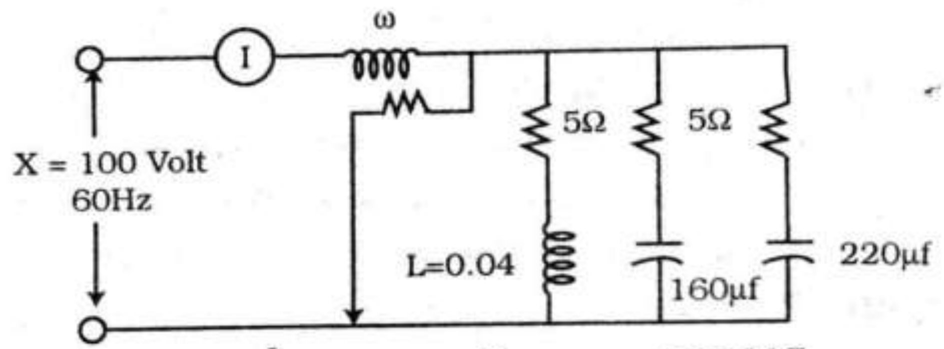
$= 100 \times 7.027 \times \cos (17.955)$

$= 668.477$ watts.

3.42 Do yourself with help of previous math

3.43 Do yourself with help of 39.

3.44



$Z_1 = 8 + \frac{-J}{2\pi \times 60 \times 220\mu}$

$= 8 - 12.057\,j$

$Z_2 = 5 + \frac{-j}{2\pi \times 60 \times 160\mu}$

$= 5 + \frac{-j}{0.0603}$

$= 5 - 16.58\,j$

$Z_3 = 5 + 15.079\,j$

$P = VI\cos\theta$

$= 100 \times 9.165 \times \cos 35.42$

$= 746.879$ watt

$Z_{total} = z_1 11 Z_2 11 Z_3$

$= (8 - 12.057j)\ 11$

$(5 - 16.58j)\ 11 (5t\ 1.079j)$

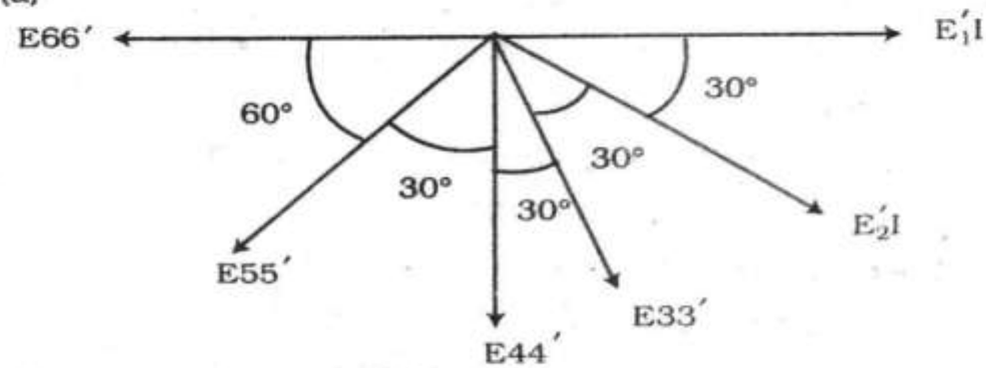
$= 10.91\angle{-35.42}$

$I = \frac{V}{Z_{total}}$

$= \frac{100}{10.91} = 9.165\angle 35.42$

3.45

(a)



(b) $E_{13}' = E_{35}' = E\angle{-60°}$

(c) $E_{13} = E\angle 0 - E\angle_{-60°}$

$= E\angle 60°$

(d) $E_{greatest} = E + E\angle{-30} + E\angle{-60} + E\angle{-90} + E\angle{-120} + E\angle{-150}$

$= (3.86\angle{-75})\ E$

(e) Do yourself

Chapter–4
Phasor Algebra
(As applied to A—C circuit Analysis)

4.1 $3 + 4j = 5\angle 53.1$

4.2 $10e^{-j120} = 10\angle -120 = -5 - j8.66$

4.3 $14\angle 60° + 20\angle 15° = 263 + j17.3$

[Tips : you can do it easily by scienfific calculator]

4.4

$A = 40\angle 120°$

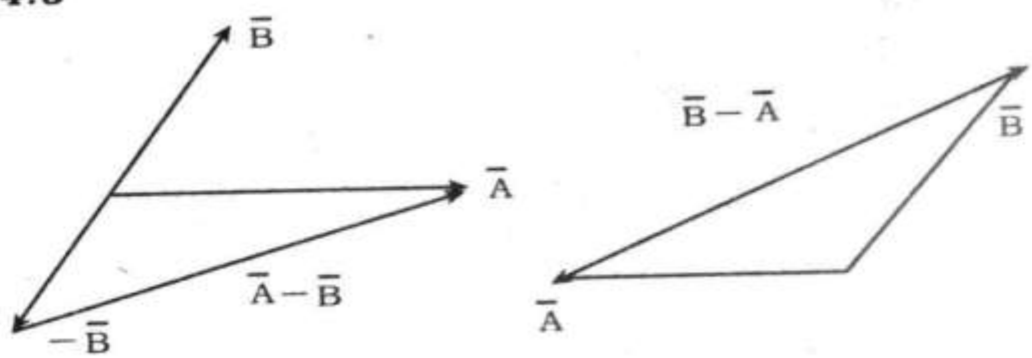
$B = 20\angle -40°$

$C = 26.46\angle 0°$

Hence,

$$A + B + C = 40\angle 120° + 20\angle -40° + 26.46\angle 0$$
$$= 21.78 + j21.78 = 30.8\angle 45°$$

4.5



4.6

$A = 42\angle 200°$

$B = 20\angle -40°$

$C = 24.25 + j14$

$\therefore \quad (A + C) - B = 32.95\angle 157.7°$

4.7

$A = 5 - 4j$

$B = 2 + 3j$

$\therefore \quad AB = (5 - 4j)(2 + 3j)$

$= 22 + j7$

$= 23.09\angle 17.65$

4.8

$A = 20 + j20$
$B = 30\angle -120°$
$C = 5 + j0$

(a) $A + B + C$
$= 20 + j20 + 30\angle -120° + 5$
$= 11.67\angle -31°$

(b) $(A + B)\,C$
$= 39.05\angle -50.2°$

(c) $ABC = (20 + j20)\,(30\angle -120)\,(5) = 4242\angle -75°$

4.9 $A = 40\angle 105°$ and $B = 5 + j8.66$

$$\frac{A}{B} = 4\angle 45°$$

4.10

$A = 20 + j20$
$B = 30\angle -120°$
$C = 5\angle 0°$

(a) $\dfrac{A + B}{C}$
$= \dfrac{5 - 5.98}{5}$
$= 1.56\angle -50.103$

(b) $\dfrac{BC}{A}$
$= \dfrac{30\angle -120°}{5}$
$= 6\angle -120°$

4.13 $A = 52\angle 70$

Here,

$$\frac{15\angle 70}{(3 - j4)} + \log_e (8 + j5)$$
$$= \frac{15\angle 70}{3 - j4} + (\log_e 9.433 \times e^{j32})$$
$$= 0.6 + j3.07$$

4.14

$K = 12.9$

$L = 0.056\ H$

$C = 78\ \mu f$

(a) $Z_{RLC} = 12.9 + j\left(\omega L - \frac{1}{\omega C}\right)$

$$= 12.9 + j\left(2\pi\ 60 \times 0.056 - \frac{1}{2\pi \times 60 \times 78 \times 10^{-6}}\right)$$

$$= 12.9 + j\ (21.11 - 34)$$

$$= 12.9 - 12.897\ j$$

(b) $I = 10\angle 30$

$V = I_{ZRLC}$

$= (10\angle 30°)\ (12.9 - 12.897j)$

$= 182.4\angle -15°$

4.15

$V_g = E_g - IZ_g$

$= (500\angle 0) - (27.1\angle 49.4°)\ (6.32\angle 71.6°)$

$= 500\angle 0° - 171.3\angle 121°$

$= 606\angle -14°$

$V_g = 606\angle -14°$

∴ total power = $V_g I \cos\theta + I_2 R + E_9 I \cos\theta = 8810$ watts

4.16

(a) $(5 + j8) + (-2 - j4)$

$= 3 + 4j$

(b) $(-12 + 6j) - (30 - 20j)$

$= -42 + 26j$

(c) $(16 - 12j)\ (-5 + 8j)$

$= 16 + 188j$

4.17

$Z_1 = 2 + 3j$

$Z_2 = 3 - 7j$

$Z_{eq} = Z_1 + Z_2$

$= 5 - 4j$

$= 6.4\angle -38.66$

4.18

(a) $100\angle -30$

(b) $100\angle -45$

(c) $100\angle -120$

4.19

(a) $8 + 6j$

$= 10\angle 36.889$

(b) $-57.36 + j81.92$

$= 100\angle 124.99$

4.20

$Z = 10\angle 30$

According to question,

$10\angle -10°$

4.21

(a) $(8 + 6j)\,(10\angle -120)\,(\cos 36.87° - j\sin 36.°)\,(0.1\, e^{j60})$

$= 10\angle -60$

(b) $\dfrac{(34.2 + j94)\,(10\angle -30)\,(30\angle 60)}{(20\angle 40°)\,(50\angle 30°)}$

$= 330\angle 30°$

4.22

(a) $\sqrt{4.5 - 7.79\,j} + \log_e 10\angle 172$

$= \sqrt{8.99}\,\angle\dfrac{-60}{2} + \log_e 10 + \log_e 10 + \log_e e^{j172}$

$= 3\angle -30° + \log_e 10 + j172$

(b) $\sqrt{\dfrac{(940 + j342)}{10_e j10}}$

$= \sqrt{\dfrac{1000.28\angle 20}{10_e j10}}$

$= \dfrac{\sqrt{1000.28}\,\angle 10}{\sqrt{10\angle 5}}$

$= \dfrac{31.63\angle 10}{3.162\angle 5}$

$= 10\angle 5$

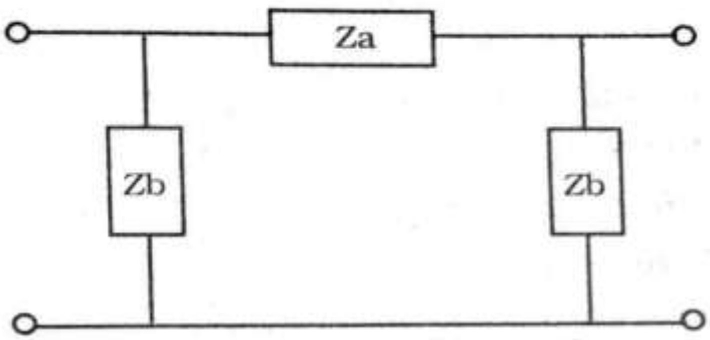
4.23

$$\sqrt[3]{\frac{10\angle 45^\circ \times 5_{ej60} \times (-4047 - j2.94)}{1 - j1.732}}$$

$$= \frac{\sqrt[3]{10}\angle 15 \times \sqrt[3]{5}\angle 30 \times \sqrt[3]{5}\ \angle -48}{\sqrt[3]{2}\angle -10^\circ}$$

In this chapter, problem no 16 + 0 23. Can be solved with calculatur directly so try it yourself.

4.24



(a) $Z_0 = \sqrt{Z_a Z_b} = \sqrt{10\angle 68 \times 25000\angle -90}$

$= \sqrt{250000\angle(-90 + 68)}$

$= \sqrt{250000\angle -22}$

$= 500\angle -\frac{22}{2}$

$= 500\angle -11$

(b) $r = \sqrt{\frac{Z_a}{Z_b}}$

[Tricks : find in polar form then,

$\sqrt{Z} = \sqrt{\text{magnitude}}\ \angle \frac{\text{Angle}}{2}$]

$= \sqrt{\frac{25000\angle -90}{10\angle 68}}$

$= \sqrt{2500\angle -158}$

$= 50\angle -79$

4.25 A voltage of 125∠40 volts

$V = 125\angle 40$

$Z = 2 + j8$

$I = \frac{125\angle 40}{2 + j8} = 15.158\angle -35.96$

4.26

$Z_1 = (1 - j3)$

$Z_2 = (3 + 6j)$

$I = 10$ amp

(a) $10\angle 0 = \dfrac{Z_2}{Z_1 + Z_2} \times 1$

$$I = \frac{(Z_1 + Z_2)}{Z_1} \times 10$$

$$= \frac{6.708\angle 63.434}{1} \times 10$$

$$= 67.08\angle 63.43$$

(b) $I_0 = I_1 + I_2$

or, $I_2 = I_0 - I_1$

$$= 67.08\angle 63.43 - \frac{10}{0}$$

$$= 20 + 60j$$

4.27

$$Z_{OT} = \sqrt{Z_1Z_2 + \frac{Z_1^2}{4}}$$

$$= \sqrt{(30\angle 86.0)\ (10\angle -90^\circ) + \frac{(30\angle 86)^2}{4}}$$

$$= \sqrt{77.16\angle 7.736}$$

$$= 8.78\angle 3.868$$

4.28

$$\sqrt{\frac{125\angle -90^\circ}{5\angle 90^\circ}}$$

$$= 1.61 \pm j\frac{\pi}{2}$$

4.29

$\alpha + j\beta$

$$= 2\log_e\left(\sqrt{1 + \frac{Z_1}{Z_1Z_2}} + \sqrt{\frac{Z_1}{4Z_2}}\right)$$

$$= 2\log_e\left(\sqrt{1 + \frac{25.14\angle -90^\circ}{795\angle 90}} + \sqrt{\frac{25.14\angle -90}{795\angle 90}}\right)$$

$$= 2\log_e (\sqrt{0.968} + \sqrt{0.03}\ \angle 90^\circ)$$

$$= 2\log_e 0.99\ ej\ \frac{9.98}{180^\circ}$$

$$\alpha + j\beta = 2 \times \log_e 0.99 + 2 \times \frac{j\,9.98}{180}$$

$$\alpha \quad = 2 \times \log_e 0.99 = 0.02$$

$$\beta \quad = 2 \times \frac{9.98}{180} = 0.11$$

4.30 Similar to 29

4.31

$V_m = V - ZI$

$V = 100\angle 0^\circ$

$Z = 15\angle 80^\circ$

$I = 10\angle -30^\circ$

$V_m = 100 - 15\angle 80^\circ \times 10\angle -3 = 160.47\angle -65.61$

4.32

(a) $(12 + a) + jb = 20 + j10$

$12 + a = 20$

$a = 8$

$b = 10$

4.33

$a + 10 = 100 \cos\beta$

$50 = 100 \sin\beta$

$\beta = \sin^{-1} \frac{50}{100} = 30^\circ$

$a + 10 = 100 \times 50\sqrt{3}$

$a = -10 + 50\sqrt{3}$

4.34

$V = 100 - j50 = 111.8\angle -26.56$

$I = -2 - j8 = 8.246\angle -104.03$

$P_{absorb} = 111.8 \times 8.246 \cos(-26.56 + 104.03)$

$= 200$ watts

4.35

$Vart = 111.8 \times 8.246 \times \sin(-26.56 + 104.03)$

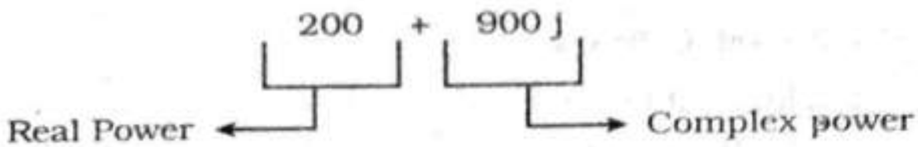
$= 900$ vars

4.36 As total power

VI

$= 111.8\angle{-26.56} \times 8.246\angle{+104.03}$

$= 200 + 900j$



4.37

$V = 40\angle 80$

$I_1 = 5\angle 30^\circ$

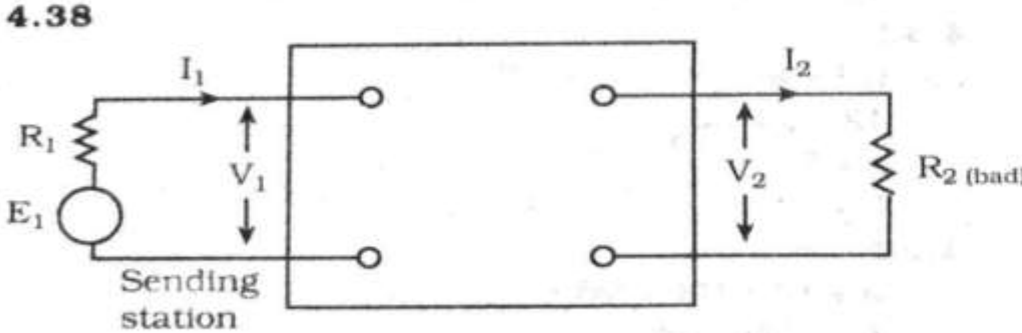
$I_2 = (-6 + j8)$

$\therefore \quad I_0 = I_1 + I_2 = 5\angle 30 + (-6 + j8) = 10.63\angle 100$

$\therefore \quad$ Complex power $= 40\angle 80 \times 10.63\angle{-100}$

$= 425.2\angle{-20} = 400 - 145j$

4.38



Here,

$R_1 = 200\Omega$

$R_2 = 20 \times 10^3\ \Omega$

$V_2 = (0.1\ \angle 114.6)\ E_1$

$$Y = \alpha + j\beta\ = \log_e \frac{Z_T}{2\sqrt{R_1R_2}}$$

$$I_{2(Opt)} = \frac{E_1}{2\sqrt{200 \times 20 \times 10}} = \frac{E_1}{4000}$$

$$I_{2(actual)} = \frac{E_1}{200 + 20 \times 10^3} = \frac{0.1\angle 14\ E_1}{20200}$$

$$\therefore \quad Y = \alpha + j\beta = \log_e \frac{Z_T}{2\sqrt{R_1 R_2}}$$

$$= \log_e \frac{I_{2(oPt)}}{I_{2(general)}}$$

$$= \log_e \frac{20200}{4000 \times 0.1\angle 114}$$

$$\alpha + j\beta = \log_e (50.5\angle -114)$$

$$= \log_e 50.5 - j\frac{114}{180}$$

$$\therefore \quad \alpha + j\beta = 3.92 - j0.633$$

4.39

$R_1 = 200$

$R_2 = 20{,}000$

$I_2 = \frac{E_1}{4000}$ amp

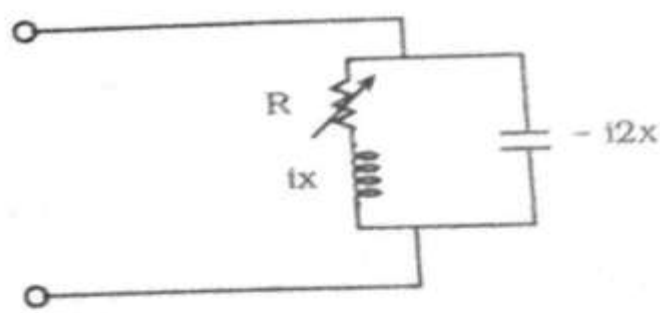
$$\alpha + j\beta = \log_e \frac{Z_T}{2\sqrt{R_1 R_2}}$$

$$= \log_e \frac{I_{2(OPt)}}{I_{2(actual)}}$$

$$= \log_e \frac{\frac{E_1}{4000}}{\frac{E_1}{4000}}$$

$$= \log_e 1 = 0$$

4.40



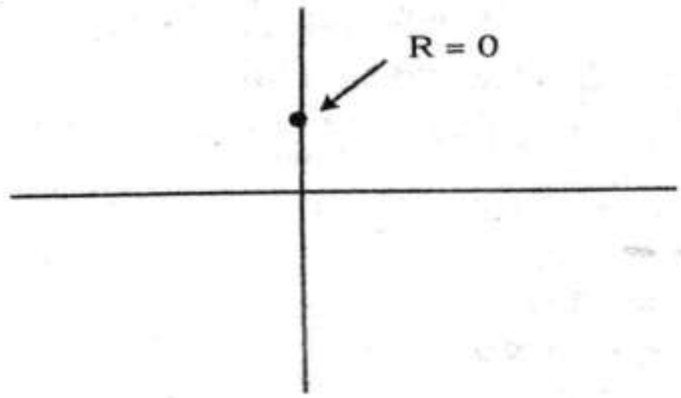
$$Z = \frac{(R + jX)(-2jX)}{R + jX - 2jX}$$

for R = 0

$$Z = \frac{(jX)(-2jX)}{jX - 2jX}$$

$$= \frac{(-2)X^2(-1)}{j(-2)X^2}$$

$$= j$$



Similarly find other values yourself.

(c) $$A\cos\omega t = A\,\frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

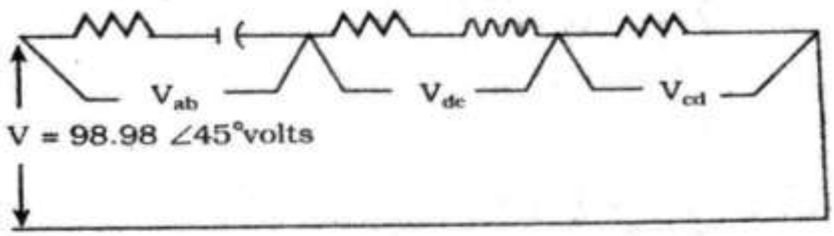
$$= \frac{A}{2}e^{j\omega t} + \frac{A}{2}e^{-j\omega t}$$

(1) is a sinusiodal circular function with a anticlockwise rotation.

(2) is a sinusodal circular function with a clockwise rotation.

Chapter–5
Sinusoidal signal – phasor circuit analysis

5.1



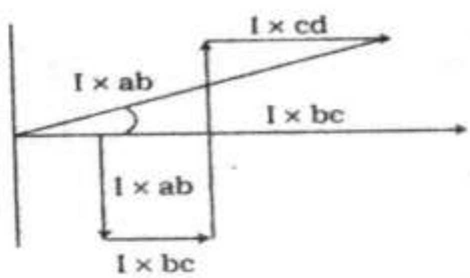
(a) $I = \frac{V}{Z} = \frac{98.98\angle 45}{2 - j4 + 3 + 11j + 2 + j0} = \frac{98.98\angle 45}{7 - j7} = 10\angle 0$

$V_{ab} = I\,Z_{ab} = (10 + j0).(2 - j4) = 44.7\angle{-63.45}$ volts

$V_{bc} = I\,Z_{bc} = (10+j0)\,(3 + j11) = 30 + j110 = 114\angle 74.75$ volts

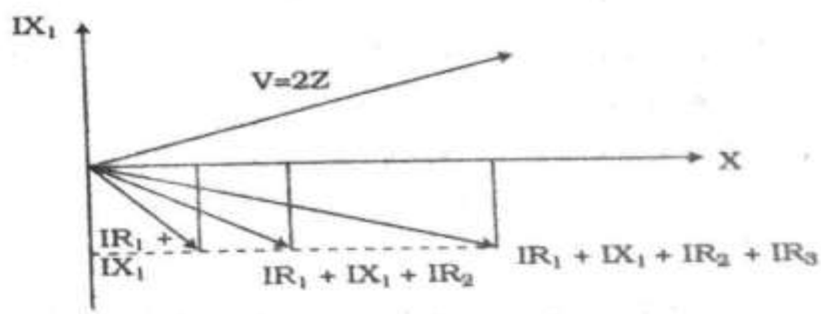
$V_{cd} = I\,Z_{cd} = (10 + j0)\,(2 + j0) = 20 + j0 = 20\angle 0°$ volts

(b)



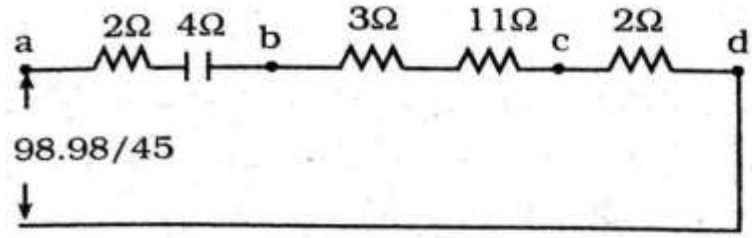
Funicular or string vector diagram of ckt.

(c)



Polar vector diagram of ckt.

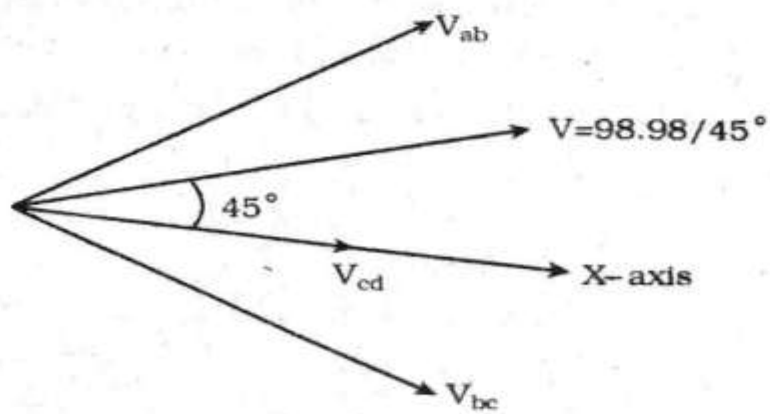
5.2



$$I = \frac{98.98\ \angle 45}{(2 - i4 + 3 + 11i + 2)}$$

$$= 9.9\ \angle 0$$

(b)



(c) Do yourself

(i) $P_R = 10^2 \times 2 + 10^2 \times 2 + 10^2 \times 3$

$= 700$ Watt

(ii) $P_R = VI \cos \theta$

$= 98.98 \times 10 \times \cos 45°$

$= 699.89 = 700$ Watt

(iii) Do yourself

5.3

$P = I^2R = 10^2 \times (2 + 3 + 2) = 700$ watts

$P = VI\cos\theta = 09/09\angle 45° \times 10\angle 0$

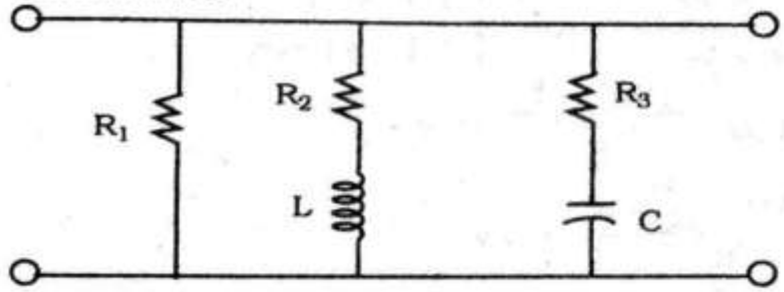
$= 989.8 \cos 45 = 99.89429 = 700$ watt

$P = vi + v'i'$

$= 69.989 \times 10 + 69.989 \times 0$

$= 699.89 \approx 700$ watts

Y = Admittance



$Y_1 = \frac{1}{Z} = \frac{1}{10 + j0} \times 0.1 + j0$ ohm

$Y^2 = \frac{1}{20 + j10} = 0.025 - j0.025$

∴ $g_2 = 0.025 \quad b_2 = 0.025$

$Y_3 = \frac{1}{30 - j40} = 0.012 + j0.016$

∴ $g_3 = 0.012, \; b3 = 0.016$

∴ Resulting $g = g_1 + g_2 + g_3$

$= 0.1 + 0.025 + 0.012 = 0.137$ ohm

$b = b_1 + b_2 + b_3 = 0 + 0.025 - 0.016 = 0.009$ ohm

$g_1 = \frac{R_1}{Z_1^2} = \frac{10}{10 + j0} = 0.1$ ohm

$b_1 = \frac{X_1}{Z_1^2} = \frac{0}{100} = 0$

$g_2 = \frac{R_2}{Z_2^2} = \frac{20}{800} = 0.025$

$b_2 = \frac{20}{800} = 0.025$

$g_3 = \frac{R_3}{Z_3^2} = 0.012$

$b_3 = \frac{-40}{2500} = -0.016$

(c) In phase component of resulting current $= V_g = 40 \times 0.137 = 5.48$ amp

Quadrature component of the resulting current

$V_b = 40 \times 0.009 = 0.36$ amp

5.4 Problems

(a) $\theta = \cos^{-1} 0.866$ | for resonance $X_L = X_C$

$= 30^\circ$ | But for p.f = 0.866

lagging | it is different

$$\angle \tan^{-1} \frac{(2\lambda f l - \frac{1}{2\pi f l})}{R} = \angle 30$$

$$\frac{2\pi f l - \frac{1}{2\pi f l}}{R} = 0.5773$$

$$\frac{X_L - X_C}{R} = 0.5773$$

$$X_L = 100 \times 0.5773 + 200$$

$$= 57.73 + 200 = 257.73$$

$$L = \frac{X_L}{2\pi f} = \frac{257.73}{377} = 0.683$$

for leading similarly,

$X_L = 142.3$ ohms and $L = 0.377$

5.5 For resonance,

$$X_C = X_L$$

$$= \frac{R^2 + X_L^2}{X_L}$$

$$= \frac{100^2 + 200^2}{200}$$

$$= 250\ \Omega$$

$$I_{max} = \frac{100}{\sqrt{100^2 + (250 - 200)^2}}$$

$$= 0.894 \text{ ampere}$$

$$\therefore\ V_{max} = 8.94 \times 250 \times 10^{-1} = 223.5 \text{ Volts}$$

5.6

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{L_C}}$$

$$= 920 \text{ KHz}$$

(b) f = Same as it depends upon L_C

(c) $f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$

and $Z = R + i\,(X_L - X_C)$

$X_L = WL = 2\pi \times 10^6 \times 150 \times 10^{-6} = 942.47$

$X_C = \frac{1}{WC} = 795.77$

(i) $Z_1 = 1 + (942.47 - 795.77)\,j = 1 + 146.69\,j$

(ii) $Z_2 = 3 + (942.47 - 795.77)\,j = 3 + 146.69\,j$

5.7

Problem-6.

(a) Draw a graph of OV where per inch voltage = 100 V

(b) Draw a graph of OC where per inch current = 20 A

(c) Draw ob, where $R_2 = 0$

$$I = \frac{V}{R + R_2 + jX}$$

$$= \frac{346.4}{2 + 0 + j\,(3.464)}$$

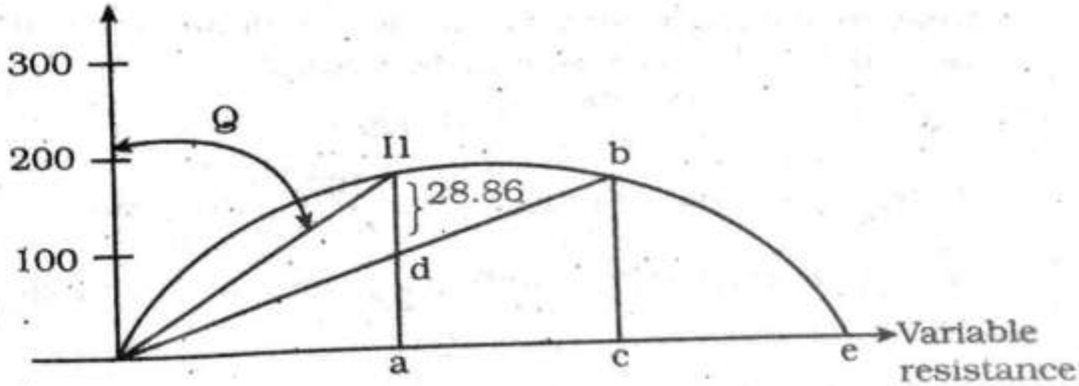
$$= 86.6 \angle -59.99$$

(d) Do yourself in graph page.

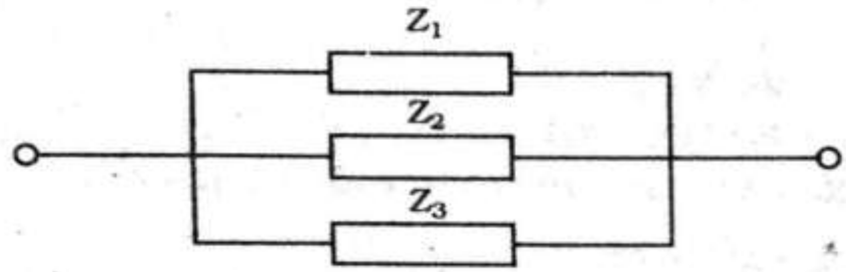
(e) $P_{max} = V \times I_1 d_{max}$

find $I_1 d_{max}$ from the graph

Then, $P_{max} = 346.4 \times 28.86$



5.8



$z_1 = 10 + j0$

$z_2 = 20 + 20\,j$

$z_3 = 30 - 40\,j$

(a) $Y_1 = \frac{1}{z_1} = 0.1$ | $g_1 = 0.1$ $b_1 = 0$

$Y_2 = 0.025 - 0.025\,j$ | $g_2 = 0.025$ $b_2 = 0.025$

$Y_3 = 0.012 + 0.016\,j$ | $g_3 = 0.012$ $b_3 = 0.016$

(b) $g = g_1 + g_2 + g_3$

$= 0.137\ \Omega^{-1}$

$b = 0.009\ \Omega^{-1}$

(c) $I = VY = V\,(g - jb) = Vg - jVb$

So $I_{\text{in page}} = vg = 5.48$ amp

$I_{\text{quadrature}} = Vb = 0.36$ amp

5.9 Given here, $R_c = 1$ ohm, $X_c = 10$ ohm $R_c = 60$ ohm

$V = 100$ v When X_L is varried, quadrature component of current in inductive branch will be maximum when it is equal to the radius of circle made by locus of it.

i.e. $I_L \sin\theta_L = \frac{V}{2R_L} = \frac{100}{2 \times 6} = 8.33$ amp

When $R_L = 4$ ohm, $I_L \sin\theta_L = \frac{V}{2R_L} = \frac{100}{2 \times 4} = 12.5$ amp

We know that, when $I_c \sin\theta_c > \frac{X}{2R_L}$ resonance cannot be obtained.

When, $I_c \sin\theta_c = \frac{V}{2R_L}$ resonance will occuronce. When I_c $\sin\theta_c < \frac{V}{2R_L}$ will occur twice

$$11R_L = 6, I_0 \sin\theta_0 = \frac{V}{\sqrt{R_0^2 + X_0^2}} \text{som} + \tan^{-1}\frac{X_c}{R_0}$$

$$= \frac{100}{\sqrt{1 + 100}} \times \sin\tan^{-1}\frac{X_c}{R_c}$$

$$= 9.95 \times 0.99$$

$$= 9.9 \text{ amps}$$

Again, $\frac{V}{2R_L}$ = 8.33 amps which is less than $I_c \sin\theta_c$

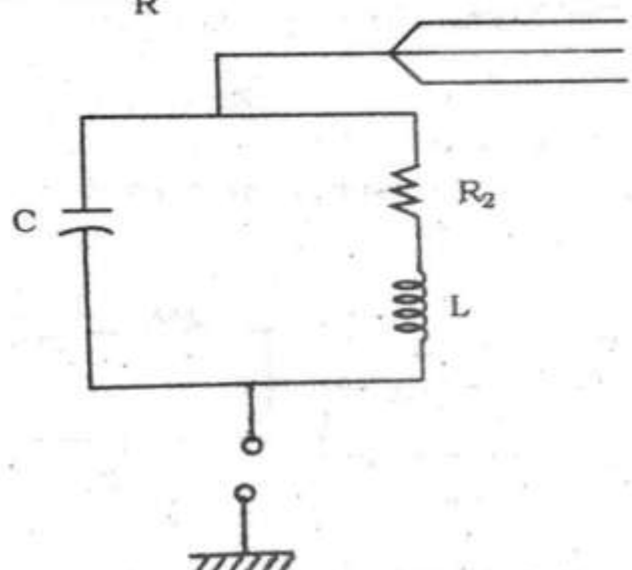
∴ resonance can be obtained when R = 4 ohm, $\frac{V}{2R_L}$ = 12.5 amps

∴ Resonance will occur at two points.

5.9

$L = 250 \times 10^{-6}$ henry

$F = 10^6$ Hz, $\frac{X_L}{R} = 170$



Now, $X_L = wL = 2\pi \times 10^6 \times 250 \times 10^{-6}$

$= 1570.8$

∴ $R = \frac{X_L}{170} = \frac{1570.8}{170} = 9.24$ ohm

(a) For resonance,

$$\frac{X_L}{R_L^2 + X_L^2} = \frac{X_c}{R_c^2 + X_c^2} \quad \left[\therefore R_c = 0 \;\; X_c = \frac{1}{\omega c}\right]$$

or, $\dfrac{X_L}{R_L^2 + X_L^2} = \omega c$

or, $C = \dfrac{1570.8}{2\pi\, 10^6 \{(9.24)^2 + (1570.8)^2\}} = 101.3 \times 10^{-12}$ f

(b) $\omega_m = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{L\omega_m^2} = \dfrac{1}{X_L\omega_m} = \dfrac{1}{1570.8 \times \omega_m}$

(c) $\dfrac{1}{CR_L} = \dfrac{250 \times 10^{-6}}{101.3 \times 10^{-12} \times 9.24} = 267090.6$ ohm

(d) $Y = 9 - jb_2 + jb_c = \dfrac{R_1}{R_L^2 + X_L^2} - j\dfrac{X_2}{R_2^2 + X_L^2} + j\omega c$

$$= \frac{9.24}{(9.24)^2 + (1555)^2} - j\frac{1555}{(9.24)^2 + (1555)^2} + j2\pi \times 990 \times 10^3 \times 101.3 \times 10^{-11}$$

$$= 3.82 \times 10^{-6} - j\,6.43 \times 10^{-4} + j\,6.3 \times 10^{-4}$$

$$= 3.82 \times 10^{-6} - j\,1.294 \times 10^{5}$$

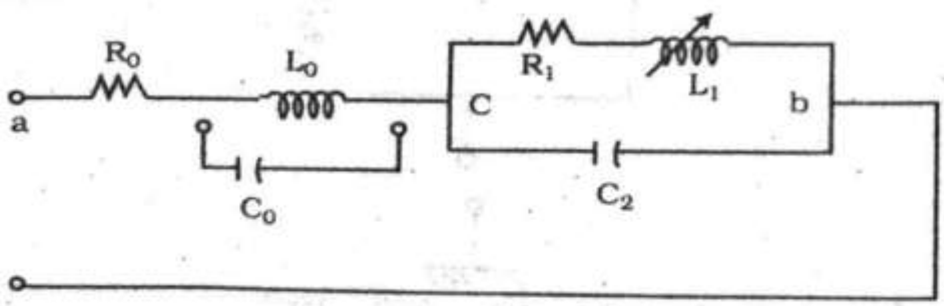
$$Z = \frac{1}{Y} = \frac{1}{3.82 \times 10^{-6} - j\,1.29 \times 10^{-5}}$$

$$= \frac{1}{1.349 \times 10^{-5} \angle -73.5}$$

$$= 74105\angle 73.5$$

5.10 Graphical measurement needed so you should try yourself.

5.11



$R_0 = 20\ \Omega$

$R_1 = 40\ \Omega$

$C_2 = 0.05\ \mu$

(a) For parallel resonance at 15000 Hz $b_z = bc$

or, $\frac{\omega L}{R_1^2 + \omega^2 L^2} = \omega c$

or, $\omega^2 L^2 C = R_1^2 C - L = 0$

or, $\omega^2 L^2 C - L + R_1^2 C = 0$

$$L = \frac{+1 \pm \sqrt{1 - 4.\omega^2.C.R_1^2 C}}{2\omega^2 C}$$

$$L = \frac{+1 \pm \sqrt{1 - 4(2\pi \times 15000 \times 0.05 \times 10^{-6} \times 40)^2}}{2 \times (2\pi \times 15000)^2 \times 0.05 \times 10^{-6}}$$

$$= \frac{1 + 0.92622}{888.264}$$

$= 2.17 \times 10^{-3}$ H (taking + ve)

or, 0.083×10^{-3} (taking – ve)

$$X_{bc} = S_{L1} = \frac{R_1}{Z^2} = \frac{40}{40^2 + (\omega L)^2}$$

$$= \frac{40}{40^2 + (2\pi \times 15000 \times 2.17 \times 10^{-3})^2}$$

∴ for lower conductance (i.e higher impedence)

we shall take 2.17×10^{-3}

(b) For 45000 Hz. $X_{L1} = 2\pi \times 45000 \times 2.17 \times 10^{-3} = 613.55$ ohm

$$X_{c2} = \frac{1}{2\pi \times 45000 \times 0.05 \times 10^{-6}} = 70.74 \text{ ohm}$$

$$\therefore \quad Z_{bc} = \frac{(40 + j613.55) \times (-j70.74)}{(40 + j613.55) + (-j70.74)} = \frac{43493.682\angle 3.73}{544.28\angle 85.78}$$

$= 79.91\angle -89.51$

$= 0.68 - j79.9$ be in predominantly capacitive

(c) An inductive reactance of 79.9 ohm should be placed in series with R_0 to produce series resonance

$$\therefore \quad L_0 = \frac{79.9}{2\pi \times 45000} = 0.283 \times b^{-3} \text{ H}$$

(d) Z_{ab} at 45000 Hz $= 0.68 - j79.9 + j79.9 + 26 = 20.68$ ohm

Z_{ab} at 15000 Hz $= 2_{bc} + Z_{ac} = \frac{1}{Y_{bc}} = Z_{ac}$

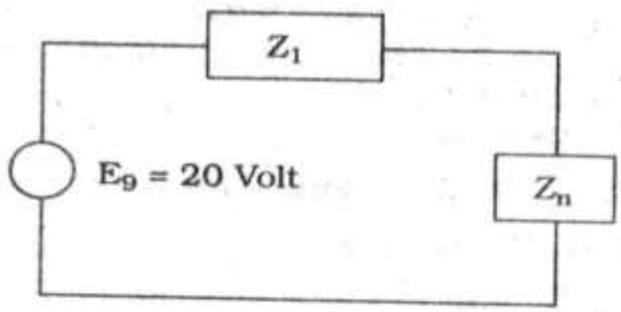
(e) $1085.687 + 20 + j2\pi \times 15000 \times 0.283 \times 10^{-3}$

$= 2205.687 + j\,26.67$

$= 1106\angle 1.38$

$\therefore\ Z_{ab} = 1106$ ohm

5.12



$Z_1 = 0.5 + j1.5 + 4j = 2 + j\,5$

$\therefore\ Z_{load} = 2 - j0.5$ ohm

$P_{max} = \dfrac{E_g^2}{4R_1} = \dfrac{2_0^2}{4 \times 2} = 50$ watts

Current (I) $= \dfrac{20}{2 + j5 + 2 - j0.5} = 5$ amps

5.13 $K = \pm \dfrac{X_r}{R_r} \pm 5$, $R_r = \dfrac{Z_1}{\sqrt{1 + K^2}} = \dfrac{\{(1.5 + 0.5)^2 + (1 + 4)^2\}^{1/2}}{\sqrt{1 + 5^2}}$

$= 1.056$ ohm

$X_r = KR_r = \pm 5 \times 1.056 = \pm 5.28$

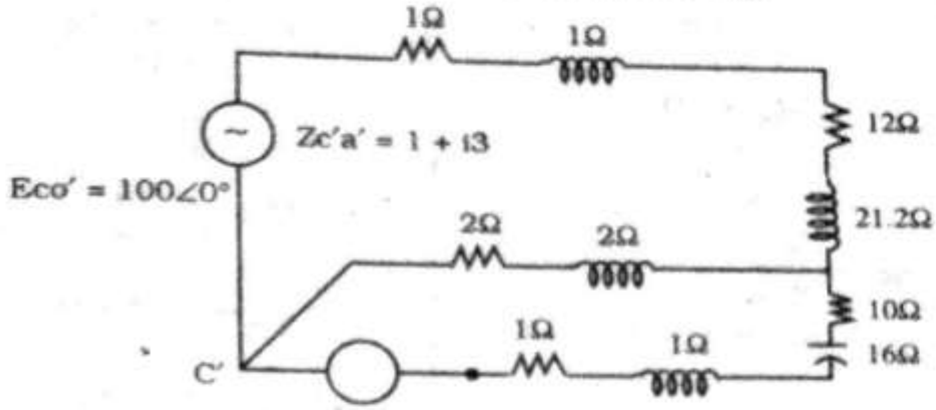
for positive K, $P_r = I^2 R_r = \dfrac{E_g^2\, R_r}{(R_1 + R_r)^2 + (X_1 + X_r)^2}$

$= \dfrac{20^2 \times 1.056}{(2 + 1.056)^2 + (5 + 5.28)^2}$

for negative. $K = P_r = \dfrac{20^2 \times 1.056}{(3.056)^2 + (5 - 5.28)^2}$

$= 44.85$

5.14 Calculate the current in branch ac of fig.



$E_{c'b'} = 100\angle 30$

$Z_{c'b'} = 1 + j3$

Assuming $E_{c'b'} = 0$, $Z_{c'a'ac} = 1 + j3 + 1 + j1 + 12 + j21.2$

$= 14 + j25.2$

$Z_{c'b'bc} = 1 + j3 + 1 + j1 + 15 - j16.6 = 17 - j12.6$

$$Z_{c'b'bcd} = \frac{(17 - 12.6j)(2 + 2j)}{17 - j12.6 + 2 + 2j} = \frac{(17 - j12.6)(2 + j2)}{(19 - j10.6)}$$

$$= \frac{21.16\angle -36.54 \times 2.82\angle 45}{21.75\angle -29.15} = \frac{59.67\angle 8.46}{21.75\angle -29.15}$$

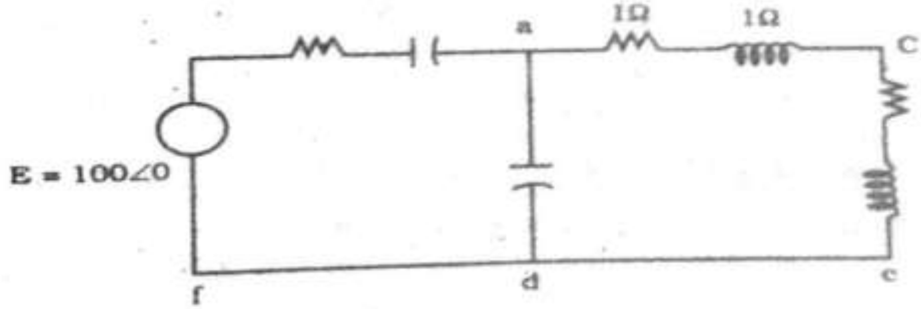
$= 2.74\angle 37.64 = 2.17 + j1.67$

$\therefore \quad Z_{a'acbc'} = 14 + j25.2 + 2.17 + j1.67 = 16.17 + j26.87$

$= 31.36\angle 58.96$

$$\therefore \quad I_{a'ac} = \frac{100\angle 0}{31.36\angle 58.96} = 3.18\angle -58.93 = 1.64 - 2.72j$$

5.15



$V_{ac} = 81 + j26$ volts

$$I_{ad} = \frac{81 + j\,26}{-j10}$$

$$= \frac{-26 + j81}{10}$$

$$= -2.6 + j8.1$$

$$I_{ef} = \frac{V_{ce}}{Z_{af}}$$

$$= \frac{(9.16\angle 76.5)\,(2.55 - j0.621)}{2 - j2}$$

$$= 8.5\angle 187.81^\circ$$

$$= -2.6 + 8.1j \text{ amp}$$

$$Z_{ec} = \frac{(2 - 2j)\,(3 + 4j)}{2 - 2j + 3 + 4j}$$

$$= \frac{6 + 8 - 6j + 8j}{5 + 2j}$$

$$= \frac{14 + 2j}{5 + 2j}$$

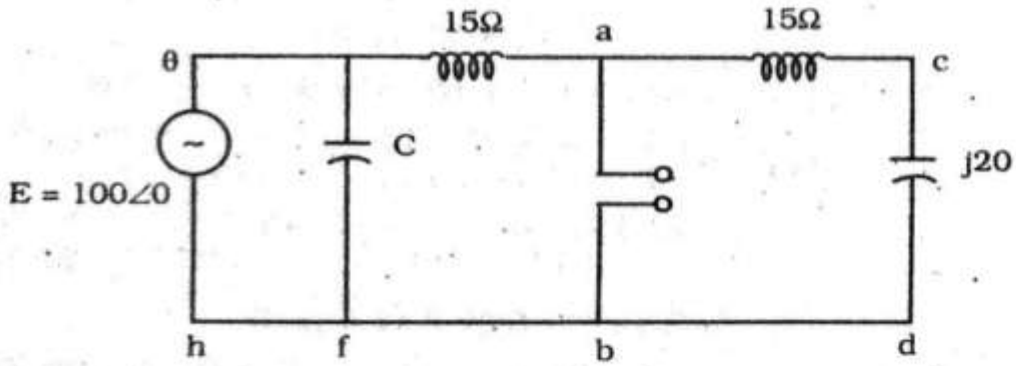
$$= \frac{(14 + 2j)\,(5 - 2j)}{25 + 4}$$

$$= \frac{1}{29}\,(70 + 4 + j10 - j28)$$

$$= \frac{1}{29}\,(7.4 - j18)$$

$$= 2.55 - j0.621 \text{ ohm}$$

5.16



$$Z_{eacd} = 5j + 5j - 20j = -10j = 10\angle -90^\circ$$

$$Z_{feacd} = \frac{10\angle -90 \times 20\angle -90}{-j10 - j20} \qquad [\therefore Z_{ef} = -20j = 20\angle -90^\circ]$$

$$= \frac{200\angle -180}{20\angle -90} = 6.66\angle -90$$

$$\therefore \text{total } I = \frac{100\angle 0}{6.66\angle -90} = 15\angle 90 = 15j$$

Assuming $E_{0'a'} = 0$

Then, $Z_{0'a'ac} = 14 + j25.2$, $Z_{cc'} = Z = 2j$

$$\therefore \quad Z_{a'acdc'} = \frac{(14 + 25.2j)(2 + 2j)}{(14 + j25.2) + (2 + 2j)}$$

$$= \frac{(14 + j25.2)(2 + 2j)}{16 + j25.2}$$

$$= \frac{81.27\angle 105.94}{31.55\angle 59.53}$$

$$= 2.57\angle 46.41 = 1.77 + 1.86j$$

$$\therefore \quad Z_{a'acbc'a'} = 17 - 12.6j + 1.77 + 1.86\,j$$

$$= 18.77 - 10.74\,j$$

$$= 21.67\angle -27.77$$

$$\therefore \quad I_{c'bc} = \frac{100\angle 90}{21.62\angle -29.97}$$

$$= 4.62\angle 119.77 = 2.29 + j4.01$$

$$\therefore \quad I_{ca2} = \frac{(2 + 2j)}{(2 + 2j) + 14 + 25.2j} \times 4.62\angle 119.77$$

$$= 0.41\angle 105.24$$

$$\therefore \quad I_{ac} = I_{a'ac1} - I_{ca2}$$

$$= 1.64 - j2.72 - (-j0.1 + j0.39)$$

$$= 1.64 - j2.72 + j0.1 - j0.39$$

$$= (1.74 - j3.11) \text{ amp}$$

$$I_{of} = \frac{100\angle 0}{-j20} = \frac{100}{20\angle -90^\circ} = 5\angle 90 = 5j$$

$$\therefore \quad I_{ea} = 15j - 5j = 10j$$

$$V_{ea} = j10 \times 5j = j^2.50 = -50 \text{ volts}$$

$$\therefore \quad \text{voltage of ab} = 100 - (-50) = 150\angle 0^\circ \text{ volts}$$

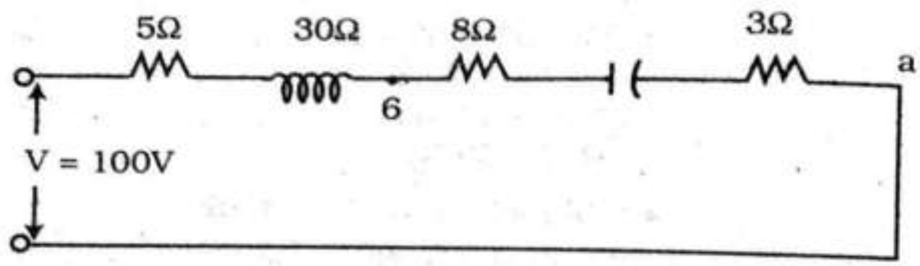
$Z_{eaf} = 5j$ [∴ when 9h point is shorted the current will not flow through ef]

$$Z_{eca} = 5j - 20j = 15\angle 90$$

$$Z_0 = (5j)\ 11\ (15j) = 7.5\,j$$

$$I_b = \frac{V_{ab}}{Z_0 + Z_L} = \frac{15}{7.5j + 10 - 7.5j} = 15\angle 0$$

5.17



$$Z = Z_{ab} + Z_{bc} + Z_{cd} = 5 + j30 + 8 - j18 + 3 + 16 + j12$$

$$\therefore \quad I = \frac{100\angle 0}{16 + j12} = \frac{100\angle 0}{20\angle 36.869} = 5\angle -36.86 = 16 - 12j \text{ amp}$$

$$V_{ab} = IZ_{ab} = 5\angle -36.869 \times (5 + 30j)$$

$$= 152 \angle 43.6686 = 109.94 + 104.95j$$

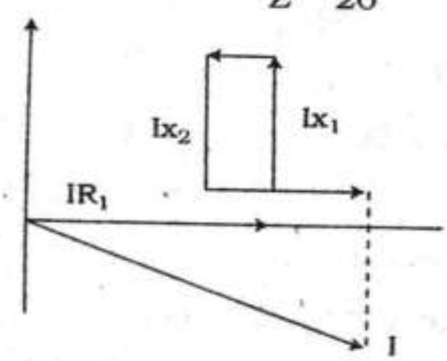
$$V_{bc} = I\,Z_{bc} = 5\angle -36.869 \times (8 - j18)$$

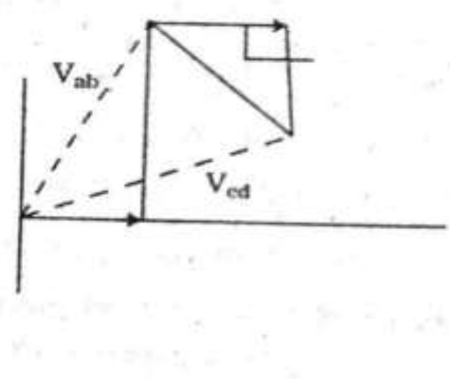
$$= 98.48\angle -102.9065$$

$$= -21.996 - j95.99 \text{ volt}$$

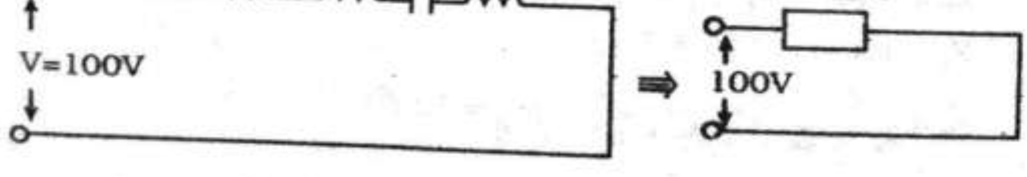
$$V_{cd} = I\,Z_{cd} = 3(16 - j12) = 48 - j36 \text{ volt}$$

$$\text{Power factor} = \frac{R}{Z} = \frac{16}{20} = 0.8 \text{ lagging}$$

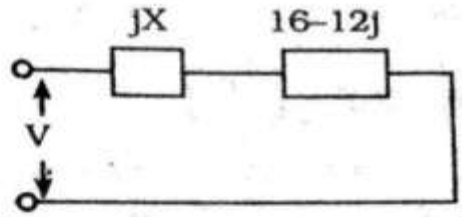




5.18



Adding a Resistance,



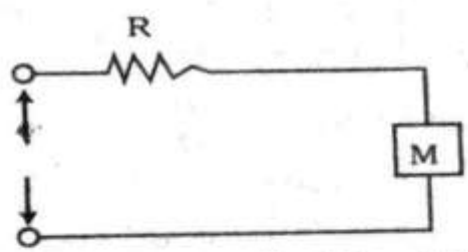
So $\tan^{-1}\frac{X-12}{16} = 0.6$

So $\frac{X-12}{16} = 0.01047$

or, $X - 12 = 0.16755$

or, $X = 12.167$

5.19



It is clear from the problem that $V_R = 110$ V

Now I_R should be deducted.

So $P = \frac{1}{4} \times 0.6 \times 746 = 111.9$ Watt

$VA = 186.5$ VA

So $I_R = I_M = \frac{186.5}{2110} = 1.695 \angle -53.13$ A

So, $R = \frac{110}{1.695} = 64.896\ \Omega$

5.20

$P = VI\cos\theta$ or $300 = 120 \times 5 \times \cos\theta \;\therefore\; \theta = 60°$

Since lagging p.f $\therefore$ voltage is leading

Now, $I^2R_L = 300$ or $R_L = \frac{300}{5^2} = 12$ ohm

Now, $\tan 60° = \frac{X_{load}}{R_{load}} \;\therefore\; X_{load} = 20.78$ ohm

$$Z = \sqrt{12^2 + (X_L - X_c)^2}$$

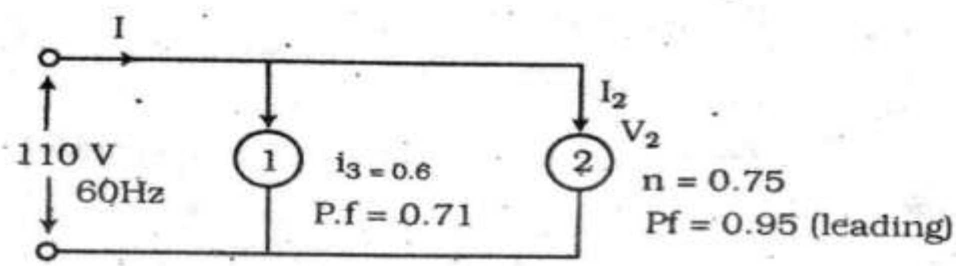
Now, $I = \frac{V}{Z}$ or, $Z = \frac{240}{\sqrt{12^2 + (X_L - X_c)^2}}$

or, $12^2 + (X_L - X_c)^2 = \left(\frac{240}{5}\right)^2$

or, $X_L - X_c = \pm\sqrt{48^2 - 12^2} = \pm 46.476$

or, $X_c = 20.78 \pm 46.476 = 67.256$

or, -25.696 (it is inductive) hence



$$P = VI_1\cos\theta = 746 \times \frac{1}{3} \times \frac{100}{60}$$

$$\therefore \quad I_1 = \frac{\left[746 \times \frac{1}{3} \times \frac{100}{60}\right]}{(110 \times 0.7)} = 5.382 \text{ amp}$$

$\cos\theta = 0.7 \therefore \theta_1 = 45.57°$

Since current is lagging

$\therefore \quad I_1 = 5.382\angle{-45.57} \qquad [\cos\theta_2 = 0.95 \therefore \theta_2 = 18.19°]$

$$I_2 = \frac{746 \times \frac{100}{75}}{110 \times 0.95} = 4.76$$

Since power factor is loading

$\therefore \quad I_2 = 4.76\angle 18.19$

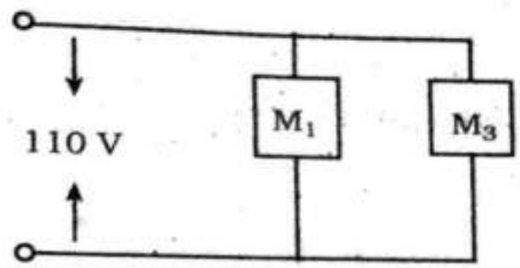
Now,

$$\begin{aligned} I &= I_1 + I_2 \\ &= 5.382\angle{-45.57} + 4.76\angle 18.19° \\ &= 3.768 - j3.843 + 4.52 + j1.49 \\ &= 8.617\angle{-15.83°} \end{aligned}$$

$P = VI\cos\theta = 110 \times 3.617 \cos(-15.83) = 911.8$ walts

Resultant power factor = $\cos(-15.83°) \doteq 0.962$ lagging

5.21



$P_1 = \frac{1}{3} \times 746 \times 0.6 = 149.2$ W

$P_2 = \frac{1}{2} \times 746 \times 0.75 = 279.75$ W

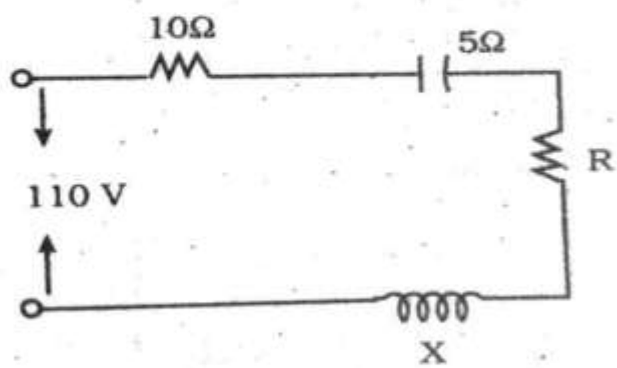
$VA_1 = \frac{149.2}{0.70} = 213.142$ VA

$VA_2 = \frac{289.75}{0.95} = 294.473$ VA

So, $I_1 = \frac{213.142}{110} = 1.937$ A

$I_2 = \frac{294.473}{110} = 2.677$ A

5.22



Now,

$I^2R = 50$

or, $I^2 = \frac{50}{R}$ ... ... (i)

$V_X = XI$

Now, $V_XI = 100$ vars

or, $XI^2 = 100$

$I^2 = \frac{100}{X}$ ... ... (ii)

From equation (i) and (2)

$$\frac{50}{R} = \frac{100}{X}$$

$\therefore \quad X = 2R$

Now, $Z = \sqrt{(100 + R)^2 + (X - 5)^2}$

Now,

$$V = ZI$$

or, $V^2 = Z^2I^2$

or, $100^2 = \{(10 + R)^2 + (2R - 5)^2\}$

or, $100^2 = (100 + 2OR + 4R^2 - 2OR + 25\} \times \frac{50}{R}$

or, $250\ R^2 - 10000R + 6250 = 0$

or, $R^2 - 4OR + 25 = 0$

$\therefore \quad R = \frac{40 \pm \sqrt{1600 - 100}}{2}$

$= 39.36$ ohm

$\therefore \quad X' = 78.72$ ohm or, 1.72 ohm

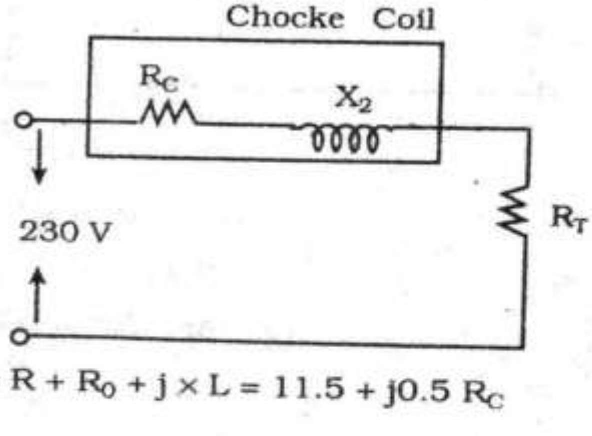
5.23

$P = VI \cos\theta$

$\therefore \quad \cos\theta = \frac{1150}{115 \times 10} = 1$

$R = \frac{V}{I} = \frac{115}{10} = 11.5$ volts

$\frac{X_L}{R_0} = 5$ i.e $X_L = 5R_C$



$Z_{CKt} = R + R_0 + j \times L = 11.5 + j0.5\ R_C$

Now.

$$I = \frac{V}{Z} \quad \text{i.e } 10 = \frac{230}{\sqrt{(11.5 + R_C)^2 + (5R_C)^2}}$$

or. $(11.5 + R_C)^2 + 25R_C^2 = 23^2$

or. $26R_C^2 + 23R_C - 396.75 = 0$

or. $R_C = \frac{-23 \pm \sqrt{529 + 41262}}{52}$

$= 3.49$ ohm

$\therefore \quad X_L = 17.45$

(a) $Z_{coil} = 3.49 + j17.45 = 17.8\angle 78.69^\circ$ ohms

(b) $Z_{CKt} = (11.5 + 3.45) + j\,17.45 = 23\angle 49.34^\circ$ ohms

P.ff = cos (49.34) = 0.652

5.24 For series resonance.

$X_L = X_C$

Reactance present in the ckt = 30 − 18 = 12 ohm

Hence, we need 12 ohm capacitive reactance.

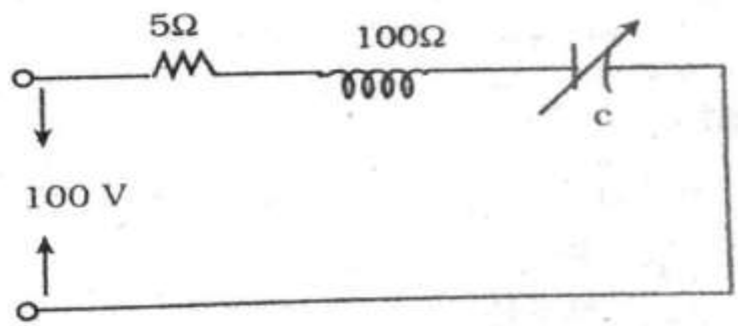
Now.

$$\frac{1}{2\pi \times 60 \times C} = 12$$

or. $C = \frac{106}{2\pi \times 60 \times 12}$

$= 221\ \mu f$

5.25



For max voltage across C

(a) $X_C = \frac{R^2 + X_1^2}{X_L} = \frac{5^2 + 100^2}{100} = 100.29$ ohm

Now,

$$I = \frac{100}{\sqrt{5^2 + (100 - 100.25)^2}} = 19.975 \text{ A}$$

Now,

$$V_C = IX_C = 19.975 \times 100.25 = 2002.5 \text{ V}$$

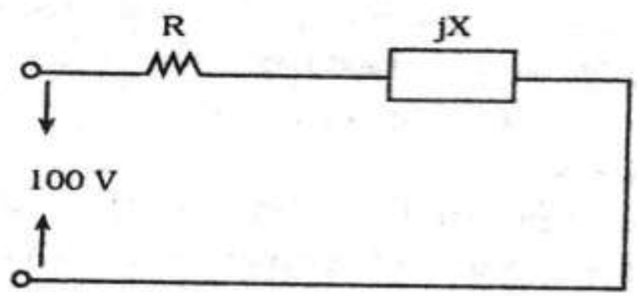
(b) $X_C = \frac{100^2 + 100^2}{100} = 200$

$\therefore$ $I = \frac{100}{100^2 + 100^2}$

$= 0.005$ A

$\therefore$ $V_C = IX_C = 200 \times 0.005 = 1$ V

5.26



$VI = 1000$

or, $100\,I = 1000$

or, $I = 10$ A

$I^2R = 800$

or, $R = 8$ ohm

$$I = \frac{100}{\sqrt{8^2 + X^2}}$$

or, $100 = \frac{100^2}{8^2 + X^2}$

or, $8^2 + X^2 = 100$

$\therefore$ $X = \pm 6$ ohm

5.27

$$Q = \frac{f_m}{\Delta f}$$

or, $\frac{f_m}{100} = 50$

$\therefore$ $f_m = 5000$ Hz

So the frequency limits on $(5000 - \frac{100}{2})$ and $(5000 + \frac{100}{2})$

i.e 4950 and 5050

5.28

(a) $f_m = \frac{1}{2\pi}$

$= \frac{1}{\sqrt{LC}}$

$= \frac{1}{2\pi\sqrt{1000 \times 10^{-3} \times 10 \times 10^{-2}}}$

$= 159154.94$ Hz

(b) $Q = \frac{1}{R_s}\sqrt{\frac{L}{C}}$

$= \frac{1}{1000} \int^{100 \times 10^{-3}} \sqrt{10 \times 10^{-12}} = 100$

(c) $Q = \frac{f_m}{\Delta f} = \frac{\omega_m}{\delta\omega}$

$\therefore \quad \Delta\omega \quad \frac{1 \times 10^6}{100} = 10^4$

$\therefore \quad \omega_m = 1 \times 10^6$ rad/see

So, $\omega_2 = (1 \times 10^6 + \frac{10^4}{2}) = 1005000$ rad/sec

$\omega_1 = (1 \times 10^6 - \frac{10^4}{2}) = 995000$ rad/sec

(d) $X_C = \frac{1}{2\pi f_C} = \frac{1}{2\pi \times 159 \times 10^3 \times 10 \times 10^{-12}}$

$= 100097.45$ 0hm

Now,

$2\pi f L = \frac{X_C^2 + R^2}{X_C}$

$\therefore \quad L = \frac{X_C^2 + R^2}{X_C \times 2\pi f} = 100.2 \times 10^{-3}$ H $= 100.2$ mH

5.29

(a) $b_L = bc$

or, $\frac{X_L}{R_L^2 + X_L^2} = \frac{X_C}{R_C^2 + X_C^2}$

or, $\frac{X_L}{X_L^2 + 31.25^2} = \frac{40}{900 + 1600}$

or, $40 X_L^2 + 2500L_L - 39002.5 = 0$

or, $X_L = \frac{-2500 \pm \sqrt{2500^2 + 4 \times 40 \times 39062.5}}{2 \times 40} = 31.29$ ohm

(b) $I_C = \dfrac{100\angle 0}{30 - j40} = 2\angle 53.13^\circ$

$OM = L_C = 2$

$MP = \dfrac{V}{2R_L} = \dfrac{100}{2 \times 31.5} = 1.6$

$\angle OMP = 180 - 53.13^\circ = 126.87^\circ$

$$OP = \sqrt{OM^2 + MP^2 - 2OM.MP\cos\angle OMP}$$
$$= \sqrt{2 + 1.6^2 - 2 \times 2 \times 1.6 \cos 126.87}$$
$$= 3.225$$

Now,

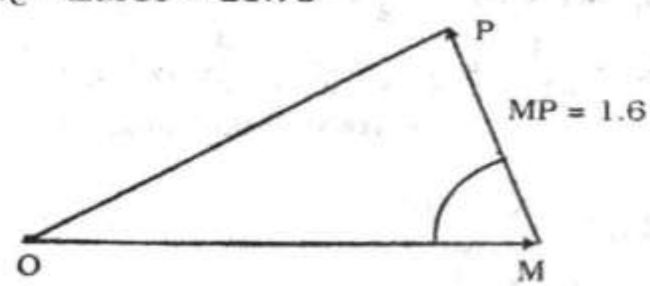
$$I_m = OP - PQ = 3.255 - 1.6 = 1.625$$

$$\therefore \quad Z_{max} = \frac{100}{1.625} = 61.54$$

Now,

$$\angle MOP = \cos^{-1}\frac{OM^2 + OP^2 - MP^2}{2OM.OP} = 23.38^\circ$$

Now, $\theta = \theta_C - \angle MOP = 29.75^\circ$



Where,

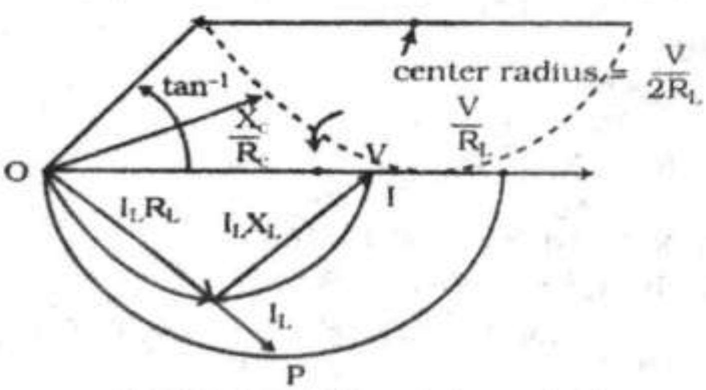
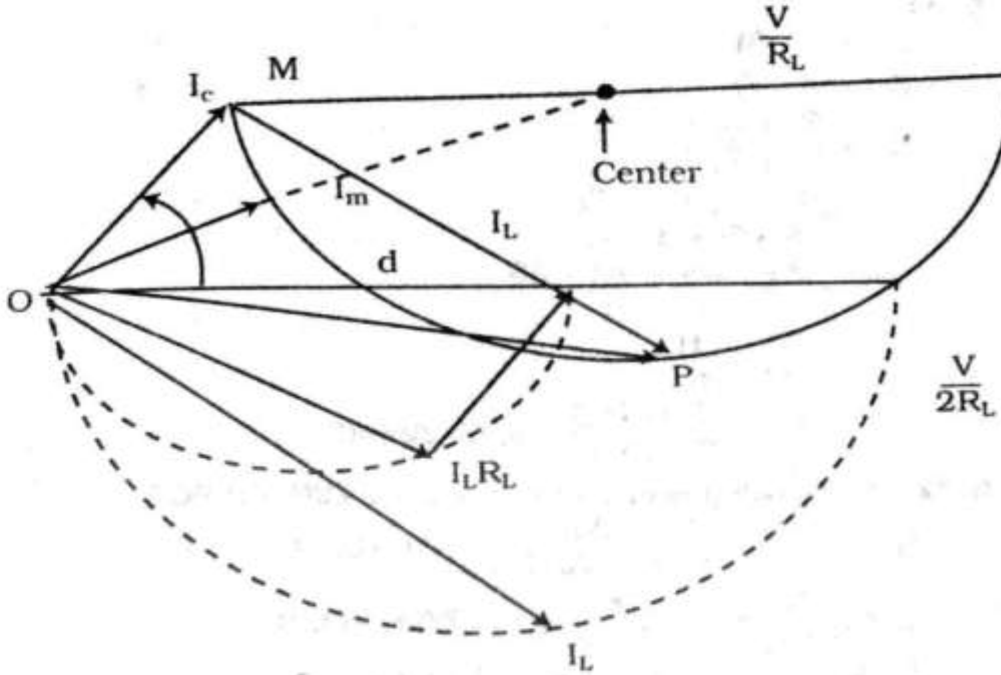
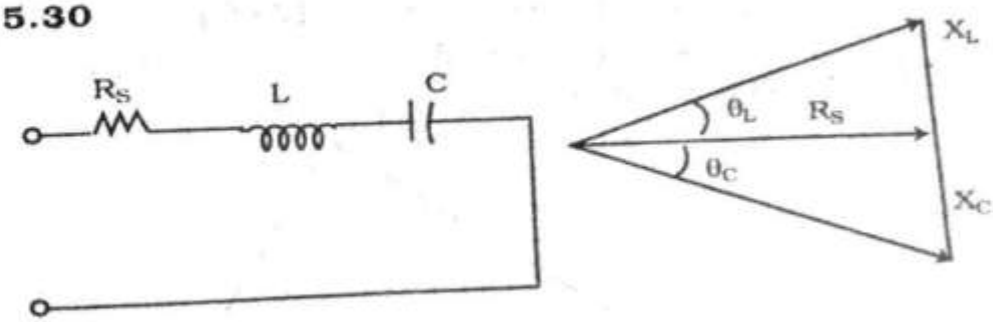


Fig : Locus of I as L is varied

Final picture when L is varied.



5.30



(a) Now,

$$\theta_s = \frac{\text{Reactive factor of the coil}}{\text{power factor of the coil}}$$

$$= \frac{\sin \theta_L}{\cos \theta_L} = \tan \theta_L$$

(b) Power factor of the coil.

$$\cos \theta_L = \frac{1}{\sec \theta_L} = \frac{1}{\sqrt{1 + \tan^2\theta_L}} = \frac{1}{\sqrt{1 + Q_S^2}}$$

where, $Q_S = \frac{X_L}{R_S} = \frac{\omega_m L}{R_S}$

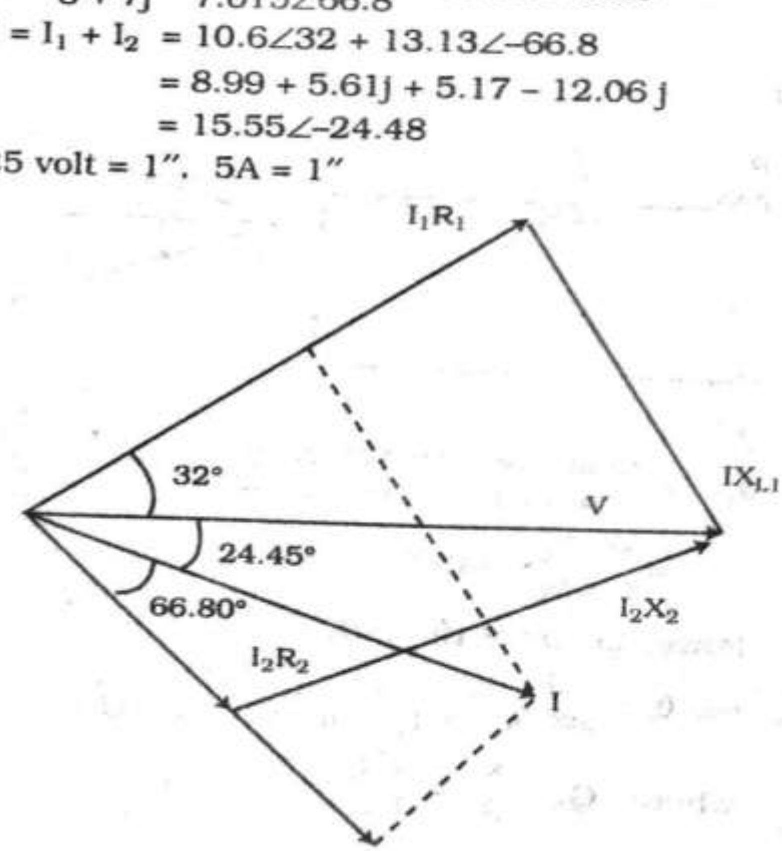
5.31

$$Z_1 = 8 - 5j$$
$$Z_2 = 3 + 7j$$
$$Z = \frac{Z_1 \times Z_2}{Z_1 + Z_2}$$
$$= \frac{(8 - 5j)(3 + j7)}{8 - 5j + 3 + 7j}$$
$$= \frac{24 - 15j + 56j + 35}{11 + 2j}$$
$$= \frac{59 + 41j}{11 + 2j}$$
$$= \frac{71.84 \angle 34.796}{11.18 \angle 10.3} = 6.43 \angle 24.45$$

5.32 $\therefore$ overall power factor = cos 24.796 = 0.907

$$I_1 = \frac{100}{8 - 5j} = \frac{100}{9.433\angle -32.605} = 10.6\angle 32$$
$$I_2 = \frac{100}{3 + 7j} = \frac{100}{7.615\angle 66.8} = 13.63\angle -66.8^\circ$$
$$I = I_1 + I_2 = 10.6\angle 32 + 13.13\angle -66.8$$
$$= 8.99 + 5.61j + 5.17 - 12.06\,j$$
$$= 15.55\angle -24.48$$

25 volt = 1″, 5A = 1″



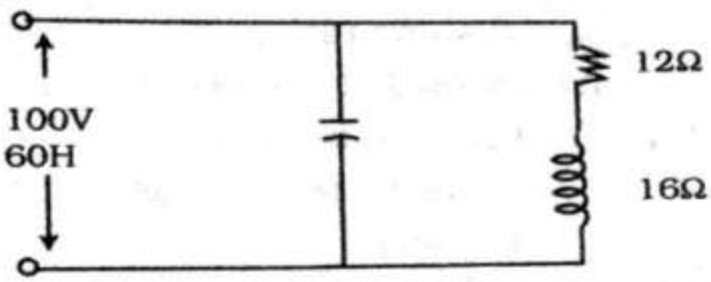
5.33 Here given,

$R_L = 12\ \Omega$

$X_L = 16\ \Omega$

$X_C = ?$

$R_C = 0$



At resonance,

$$\frac{X_L}{R_L^2 + X_L^2} = \frac{X_C}{R_C^2 + X_C^2}$$

$$\frac{16}{R^2 + 16^2} = \frac{X_C}{0^2 + X_C^2} = \frac{1}{X_C}$$

$$\therefore \quad X_C = \frac{12^2 + 16^2}{16} = 25 \text{ ohm}$$

$$\text{or,} \quad \frac{1}{2\pi f_C} = C$$

$$\therefore \quad C = \frac{1}{50\pi} \times 10^6 = 106.1\ \mu f$$

5.34

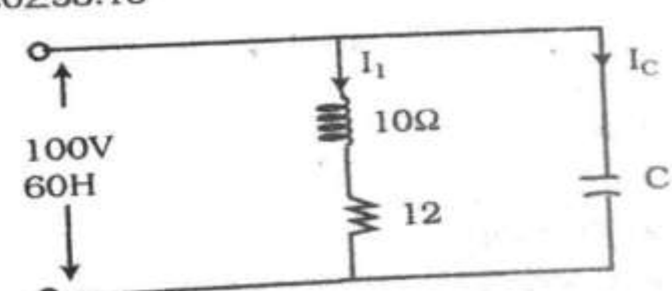
(1) When P.f = 0.8 lagging

$\theta = \cos^{-1} 0.8 = 36.86$ lagging

$Z_L = 12 + j16 = 20\angle 53.13^\circ$

$X_C = 0 - jX_C = X_C\angle -90^\circ$

$$\therefore \quad I_L = \frac{10\angle 0}{20\angle 53.13} = 5\angle -53.13^\circ$$



$$I_C = \frac{100\angle 0}{X_C\angle -90^\circ} = \frac{100\angle 90^\circ}{X_C}$$

From the vector we get,

$I_{XL} = I_L \cos 53.15° = 3$

$IYL = I_L \sin 53.15° = 40$ omp

$I' = I_{XL} \tan 36.88°$

$= 3 \times \tan 36.88° = 2.2508$

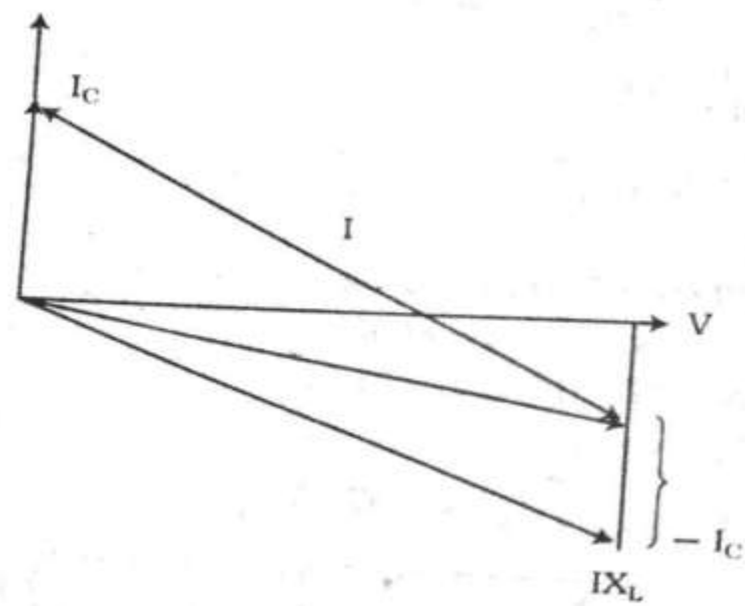
$= 1.75$ amp

$\therefore \quad |X_C| = \frac{100}{1.75} = 57.143$

$\therefore \quad C = \frac{1}{2\pi f X_C} \times 10^6 = 46.42\ \mu f$

When, $P.f = 0.8$ leading

$\theta = \cos^{-1} 0.8 = 36.87°$ leading



In this case

$I_C = I_{YL} = I'$

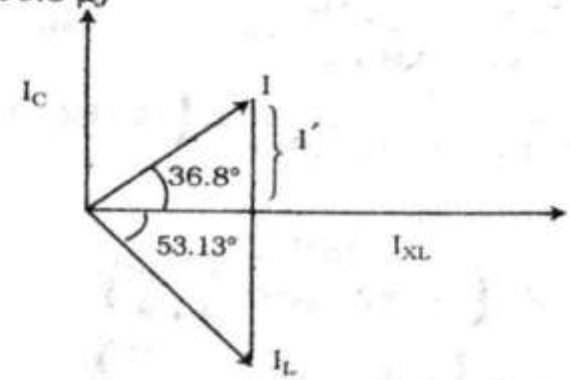
$= 4 + 2.2508$

$= 6.2508$ amp

$\therefore X_C = \frac{100}{6.2508} = 15.991$

$\therefore\ C = \dfrac{1 \times 10^6}{2 \times 3.14 \times 60 \times 15.991}$

$= 156.8\ \mu f$



5.35

$Y_1 = 0.03 - j0.04$

$Y_2 = \dfrac{1}{R}$

$Y = 0.03 + \dfrac{1}{R} = j0.04$

Impedence angle,

$\theta\ = -\tan^{-1} \dfrac{-0.04}{0.03 + \dfrac{1}{R}}$ [P.f cosθ = 0.8 θ = 36.86°]

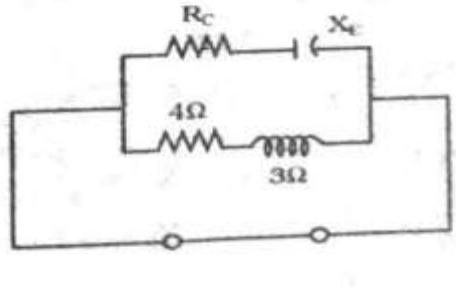
$= 36.86 = \tan^{-1} \dfrac{-0.04}{0.03 + \dfrac{1}{R}}$

or, $\dfrac{-0.04}{0.03 + \dfrac{1}{R}} = -0.75$

or, $0.0225 + 0.75\dfrac{1}{R} = 0.04$

or, $\dfrac{1}{R} = \dfrac{0.04 - 0.0225}{0.75}$ [∴ R = 42.86 Ω]

5.36



$X_C = 5$

$\therefore\ X_C = 5R_C$

$$Z = \frac{(4+3j)(R-5Rj)}{4+j3+R-5Rj}$$
$$= \frac{(5\angle 36.87^\circ)(5.1R\angle -78.69^\circ)}{\sqrt{(4+R)^2+(3-5R)^2}} \quad \angle \tan^{-1}\frac{3-5R}{4+R}$$
$$= \frac{25.5R}{\sqrt{(4+R)^2+(3-5R)^2}} \angle\left(-41.82 - \tan^{-1}\frac{3-5R}{4+R}\right)$$

Now,

$$I\angle\theta = \frac{V\angle 0^\circ}{Z\angle\left(-41.82 - \tan^{-1}\frac{3-5R}{4+R}\right)}$$
$$= \frac{V}{Z}\angle\left(41.82 + \tan^{-1}\frac{3-5R}{4+R}\right)$$

$\therefore \quad \theta = 41.82 + \tan^{-1}\frac{3-5R}{4+R}$

Now, $\theta = \cos^{-1} 0.8 = 36.87$ (since leading)

$\therefore \quad 36.87 = 41.82 + \tan^{-1}\frac{3-5R}{4+R}$

or, $\frac{3-5R}{4+R} = \tan^{-1}(-4.85)$

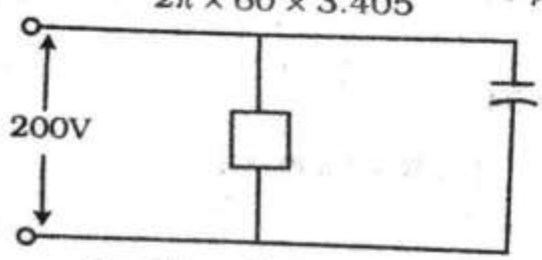
or, $3 - 5R = 0.0866(4+R)$

$\therefore \quad R = 0.681$ ohm

Now,

$X_C = 5 \times 0.681 = 3.405$ ohm

$\therefore \quad C = \frac{1}{2\pi \times 60 \times 3.405} = 779\ \mu f$



$P = VI\cos\theta$

or, $5 \times 10^3 = 200 \times I \times 0.6$

$\therefore \quad I = 41.67$ A

$\theta = \cos^{-1} 0.6 = 53.13$

Now,

$I = 41.67\angle -53.13$

Now,

$I^2R = P$

or, $R = \frac{5 \times 10^3}{41.67^2}$

$= 2.80$ ohm

Now,

$41.67 = \frac{200}{\sqrt{2.88^2 + X_L^2}}$

or, $X_L^2 = \frac{200^2}{41.67^2} - 2.88^2$

$\therefore \quad X_L = 3.81$ ohm

Now, for P.f = 1 we have

$b_L = b_C$

or. $\frac{3.84}{2.88^2 + 3.84^2} = \frac{1}{X_C}$

$\therefore$ $X_C = 6$ ohm

Now, volt – amp

$= V_C I_C = \frac{V_C^2}{X_C} = \frac{200^2}{b}$

$= 6.67 \times 10^3$ volt amp

= 6.67 KVA

5.37

5.38

5.39

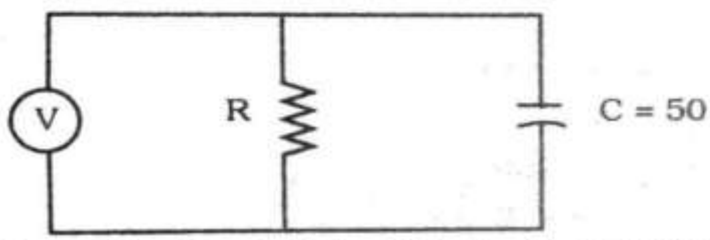
5.40 $KVA = \frac{K\omega}{P.f} = \frac{2000}{0.8} = 2500$

Required cost = \$ 2500 × (200 × 0.08) = \$40,000

$KVA = \frac{2000}{1} = 2000$

$\therefore$ Required cost = 200 × 200 × 0.08 = \$32000

5.41



Here given. C = 50 μf. P.f = 0.6. R = ? f = 60 Hz

We know that.

$X_C = \frac{10^6}{2\pi fC\mu f} = \frac{10^6}{2\pi \times 60 \times 50}$

= 53.05 ohm

Here, $\cos\theta = 0.6$ $\theta = \cos^{-1} 0.6 = 53.13°$

$\tan\theta = \frac{X_C}{R}$

$\therefore$ $\tan(53.13) = \frac{53.05}{R}$

$\therefore$ R = 39.78 ohm

5.42

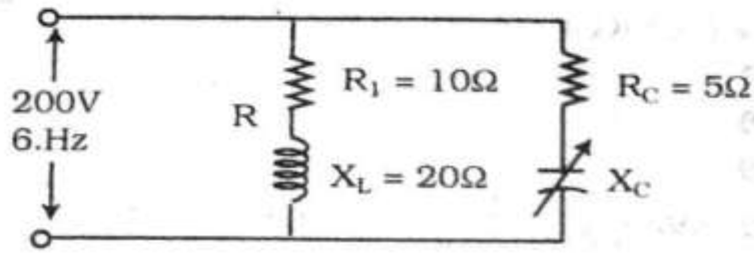
$$f_m = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{100\times10^6\times400\times10^{-12}}}$$

$$= 795.775\times10^3\text{ Hz}$$

5.43. For resonance

$$b_L = bc$$

$$\frac{20}{10^2+20^2} = \frac{X_C}{X_C^2+5^2}$$



or. $X_C^2 - 2500\,X_C = 25\times20 = 0$

or. $X_C^2 - 25\,X_C = 25 = 0$

$\therefore\quad X_C = \dfrac{25\pm\sqrt{25^2-4.25}}{2}$

$= 23.96$ ohm

or. 1.044 ohm

$$|I_C| = \frac{200}{\sqrt{5^2+23.96^2}} = 8.172\text{ A}$$

$\therefore\quad P_C = 8.172^2\times5 = 333.9\ \omega$

$$|I_C| = \frac{200}{\sqrt{5^2+1.044^2}} = 39.156\text{ A}$$

$\therefore\quad P_C = 7665.96\ \omega$

5.44 For max^m impedance current will be min.

$$I_L = \frac{200\angle0}{10+j20}$$

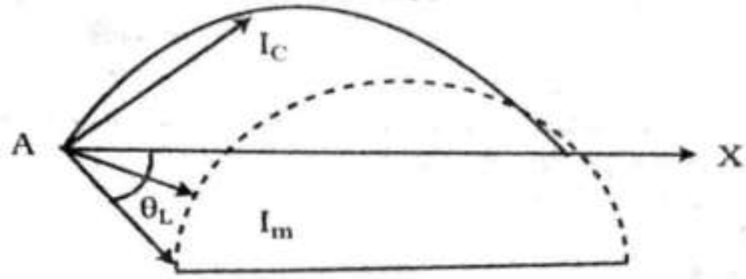
$$= 8.944\angle{-63.43^\circ}$$

For triangle ABP

$AB = |I_L| = 8.94$

$BP = \frac{V}{2R_C} = \frac{200}{2 \times 5} = 20$

$\angle ABP = 180° - 63.43 = 16.57°$



Now,

$AP = (AB^2 + BP^2 - 2AB.BP \cos\theta\angle ABP)^2 = 25.3$

$|I_m| = 25.3 - 20 = 5.3A$

$\angle PAB = \cos^{-1} \frac{AB^2 + AP^2 - BP^2}{2AB.AP} = 44.97°$

$\therefore \quad \angle PAX = 63.43 - 44.97 = 18.46°$

$\therefore \quad I_m = 5.3 \angle -18.46°$

Now,

$I_C = I_m - I_L$

$= 5.3\angle -18.46 - 8.94\angle -63.43°$

$= 1.03 + j6.32$

$= 6.4\angle 80.74$

Now,

$6.4 = \frac{200}{\sqrt{5^2 + X_C^2}}$

or. $|X_C| = 30.85$ ohm

$\therefore \quad C = \frac{1}{2\pi \times 60 \times 30.85} = 85.98\ \mu f$

5.45 From the circle diagram, there will be only one resonance.

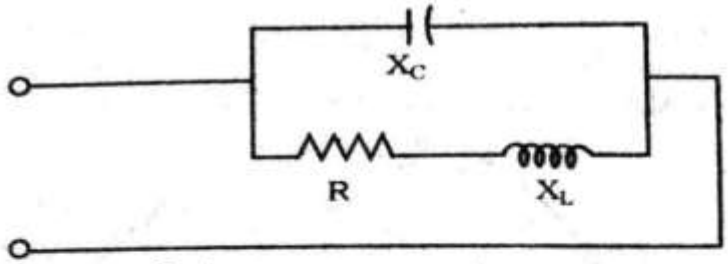
when.

$I_L \sin\theta_L = \frac{V}{2R_C}$

or. $R_C = \frac{200}{18.94 \sin 63.43} = 12.506$ ohm

Hence any value of R_C over 12.506 ohm will prevent the possibility of attaining resonance.

5.46.



For unity power factor

$b_L = b_C$

$$\frac{X_L}{R^2 + X^2} = \frac{1}{X_C}$$

$$X_L^2 - X_L X_C + R^2 = 0$$

$$X_L = \frac{X_C \pm \sqrt{X_C^2 - 4R^2}}{2}$$

$$= \frac{X_C}{2} \pm \sqrt{\frac{X_C}{4} - R^2}$$

5.47

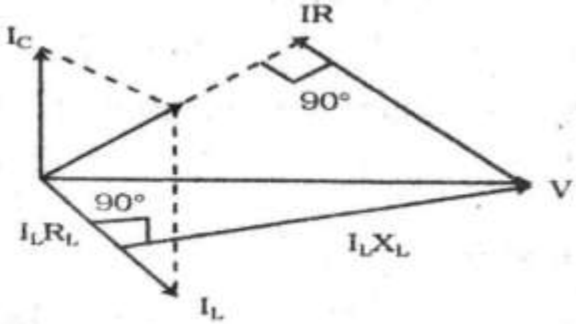
$$Y = j\omega c + \frac{1}{R}$$

$$Y = \sqrt{\frac{1}{R} + (\omega c)^2} \angle \tan^{-1} \frac{\omega c}{\frac{1}{R}}$$

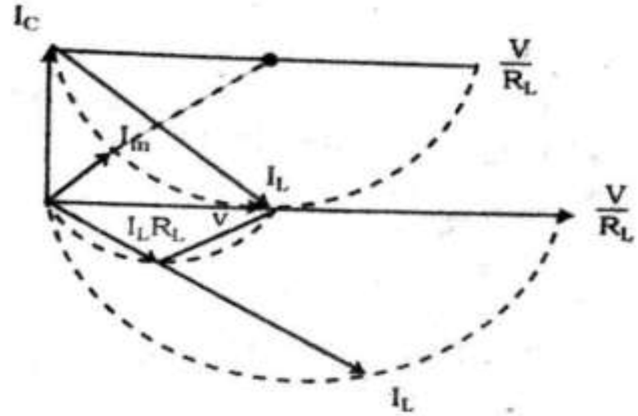
$$Y = f = \frac{\omega}{2\pi}$$

Plot the graph (semi-log graph of which X is used as log scale) of given values.

5.48 (a) Vector diagram.



(b)



when X_L is swept from 0 to ∞

(c) Analylically.

$b_L = bc$

$$\frac{X_L}{R^2 + X_L^2} = \frac{1}{X_C}$$

or, $$\frac{X_L}{64 + X_L^2} = \frac{1}{20}$$

or, $X_L^2 - 20X_L + 64 = 0$

or, $$X_L = \frac{20 \pm \sqrt{20^2 - 4 \times 64}}{2}$$

$$= \frac{20 \pm 12}{2}$$

$= 10 \pm 6$

$= 16\Omega$ or 4Ω

(d) $I_C = \frac{120}{20} = 60\angle{-90^\circ}$

$I_L = \frac{120}{2 \times 16} = 3.75\angle 90^\circ$ [neglecting R = 8 ohm]

$OP = \sqrt{6^2 + 3.75^2 - 2.6.3.75 \cos(90 + 90^\circ)}$

$= 4.684$

$\therefore$ Minimum current,

$i_m = 6 - 4.684$

$= 1.316$ amp

we have assumed that $X_L = 16$ for $i_m = 1.316$ amp

5.49

When, $R_L = R_C$

(a) $f = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{0.05 \times 200 \times 10^{-6}}} = 50.33$ Hz

(b) $\dfrac{X_C}{R_C^2 + X_C^2} = \dfrac{X_L}{R_L^2 + X_L^2}$

or, $R_C^2 = \dfrac{X_C\,(R_L^2 + X_L^2)}{X_L} - X_C^2$

$= \dfrac{35.37\,(20^2 + 14.14^2)}{14.14} - 35.37^2$

$\therefore$ $R_C = 15.8$ ohm

(c) For parallel resonance, irrespective of frequency

$R_L = R_C = \sqrt{\dfrac{L}{C}}$

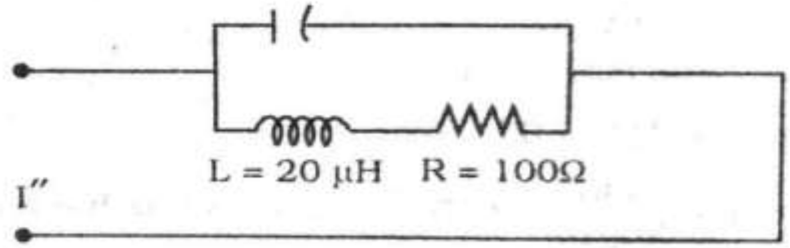
or, $30^2 = \dfrac{L}{100 \times 10^{-6}}$

$L = 0.04$ Henry

5.50

$X_L = 5\times10^7\times20\times10^{-6} = 1000\ \Omega$

$X_C = \dfrac{1}{5\times10^7\times20\times10^{-12}} = 1000\ \Omega$



$g_L = \dfrac{100}{100^2 + 1000^2} = 9.9 \times 10^{-5} = 0.0001$

$b_L = \dfrac{1000}{100^2 + 1000^2} = 9.9 \times 10^{-4} = 0.001$

$bc = \dfrac{1}{X_C} = \dfrac{1}{1000} = 0.001$

(b) $V = IZ = 2 \times 10^{-3} \times 10049.9\angle{-5.7} = 20.09\angle{-5.7}$

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2} = 10019.9\angle{-5.7^\circ}$$

(c) $OP = \frac{1}{g}\sqrt{\frac{C}{L}} = \frac{1}{0.0001}\sqrt{\frac{20 \times 10^{-12}}{20 \times 10^{-6}}} = 10$

(d) $$Z = \frac{Z_L Z_C}{Z_L + Z_C}$$

$$= \frac{(R + j\omega L)\left(-j\frac{1}{\omega C}\right)}{R + j\omega L - \frac{j}{\omega C}}$$

$$= \frac{\frac{L}{C} - j\frac{R}{\omega C}}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$= \frac{\left(\frac{L}{C}\right) - \frac{jR}{\omega C}\left\{R - j\left(\omega L - \frac{j1}{\omega C}\right)\right\}}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\therefore \quad R_Z = \frac{\frac{RL}{C} - \frac{R}{\omega C}\left(\omega L - \frac{1}{\omega C}\right)}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$= \frac{\left(\frac{RL}{C} - \frac{RL}{C} + \frac{R}{\omega^2 C^2}\right)\omega^2 C^2 R}{R^2\omega^2C^2 + (\omega^2 LC - 1)\ R^2\omega^2C^2 + (\omega^2 LC - 1)}$$

5.52

$$R_z = \frac{R}{R^2\omega^2C^2 + (\omega^2LC - 1)^2}$$

R_Z ill be max when the denominator of the above expression will be max^m

Now,

$$\frac{do}{d\omega} = \frac{d}{d\omega}\{R^2\omega^2C^2 + (\omega^2LC - 1)^2\} = 0$$

or, $2\omega R^2\omega + 2.2\omega LC\ (\omega^2LC - 1)\} = 0$

$\therefore \quad \omega = 0$. But then line voltage will be dc

Again,

$$R^2C^2 + 2LC(\omega^2LC - 1) = 0$$

or, $R^2C + 2L(\omega^2LC - 1) = 0$

or, $\omega^2 = \frac{2L - R^2C}{2L^2C}$

or, $\omega = \frac{2L}{2L^2C} - \frac{R^2C}{2L^2C}$

or, $\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$ rad/sec

(b) $\omega = \sqrt{\frac{1}{20 \times 10^{-6} \times 20 \times 10^{-12}} \times \frac{100^2}{2(20 \times 10^{-6})^2}}$

$= 4.9875 \times 10^7$ rad/sec

(c) $\omega = \frac{1}{\sqrt{LC}} = 5 \times 10^7$ rad/sec

5.53 $R_{zmax} = \frac{R}{(LC\omega^2 - 1) + R^2\omega^2C^2}$

$= \frac{100}{(20\times10^{-6}\times20\times10^{-12}\times4.9875^2\times10^{14}-1)^2+(100\times4.98875\times10^7\times20\times10^{-12})}$

$= 10025.06$

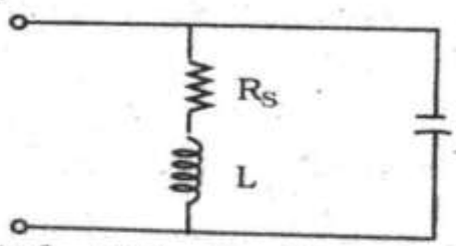
5.54 $\omega = \frac{0.1}{\sqrt{LC}} = \frac{0.1}{\sqrt{20 \times 10^{-6} \times 20 \times 10^{-12}}} = 5 \times 10^7$

$Q_P = \frac{1}{9\omega L} = \frac{1}{0.0001 \times 5 \times 10 \times 20 \times 10^{-6}} = 10$

5.55 $i_L = \frac{V}{R_S^2 + (\omega_m L)^2}$

Since, $R_S^2 << \omega_m^2 L^2$

$\therefore \quad i_L = \frac{V}{\omega_m L}$



given that, $\omega = \frac{Li_L^2}{2} + \frac{CV^2}{2} = \frac{L}{2} \cdot \frac{V^2}{\omega_m^2 L^2} + \frac{CV^2}{2}$

Again,

$$\omega_m = \frac{1}{\sqrt{LC}} \text{ (given)}$$

or, $\frac{V^2}{2\omega_m{}^2L} + \frac{CV^2}{2}$

or, $\frac{CV^2}{2} + \frac{CV^2}{2} = \omega$

$\therefore \quad C = \frac{\omega}{V^2}$

Now, given that

$$Q_P = \frac{\omega_m C}{g}$$

$$= \frac{\omega_m}{g} \times \frac{\omega}{V^2}$$

$$= \frac{\omega_m \omega}{V^2 g}$$

5.56

(a) $R_C = 10$ ohm

$$\therefore \quad g_C = \frac{R_C}{R^2 + X_C{}^2}$$

$$= \frac{R_C}{R_C{}^2 + \frac{1}{\omega^2 C^2}}$$

$$= \frac{R_C}{R_C{}^2 + \frac{LC}{C^2}}$$

$$= \frac{R_C}{R_C{}^2 + \frac{L}{C}}$$

$$= \frac{10}{100 + \frac{20 \times 10^6}{20 \times 10^{-12}}}$$

$$= 9.999 \times 10^{-6} \text{ mho}$$

$\therefore$ So equivalent parallel resistance of two capacitor is

$$\frac{1}{g_C} = \frac{1000}{0} \text{ ohm}$$

(b) Now,

$$g_L = \frac{100}{10000 + \omega m^2 L^2}$$

$$= \frac{100}{10000 + \frac{1}{LC} \times L^2}$$

$$= \frac{100}{10000 + \frac{L}{C}}$$

$$= \frac{100}{10000 + \frac{20 \times 10^{-6}}{20 \times 10^{-12}}}$$

$$= 9.9 \times 10^{-5} \text{ mho}$$

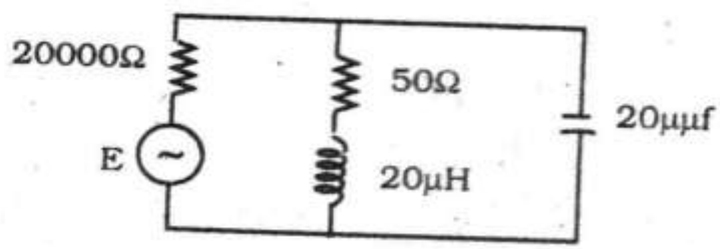
$\therefore$ E_q parallel resistance of the inductor $\frac{1}{g_L} = 10100$ ohm

Hence, eq. resistance of the parallel ckt is

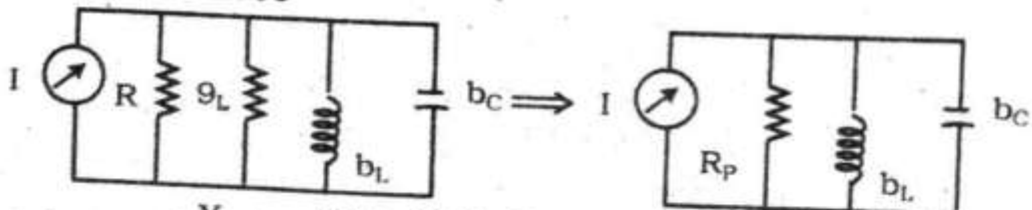
$$\frac{100010 \times 10100}{100010 + 10100}$$

$= 9173.56$ ohm

5.57



$$= \frac{V}{20000}$$



$$b_L = \frac{X_L}{R^2 + X_L^2} = \frac{20 \times 10^{-6} \times 5 \times 10^7}{50^2 + (1000)^2} = \frac{1000}{1002500}$$

$$= 9.975 \times 10^{-4} \text{ mho}$$

$b_C = \frac{1}{X_C} = \frac{1}{5 \times 10^7 \times 20 \times 10^{-12}} = 0.001$ mho

$R = 20{,}000$

$g_L = \frac{50}{50^2 + (1000)^2} \cdot R_L = \frac{1}{g_L} = \frac{50^2 + 1000^2}{50} = 20050$ ohm

Now, $R_P = \frac{20000 \times 20050}{20000 + 20050} = 10012.5$ ohm

(b) $g_P = \frac{1}{g}\sqrt{\frac{L}{C}} = 10012.5\sqrt{\frac{20 \times 10^{-12}}{20 \times 10^{-6}}} = 10.01$

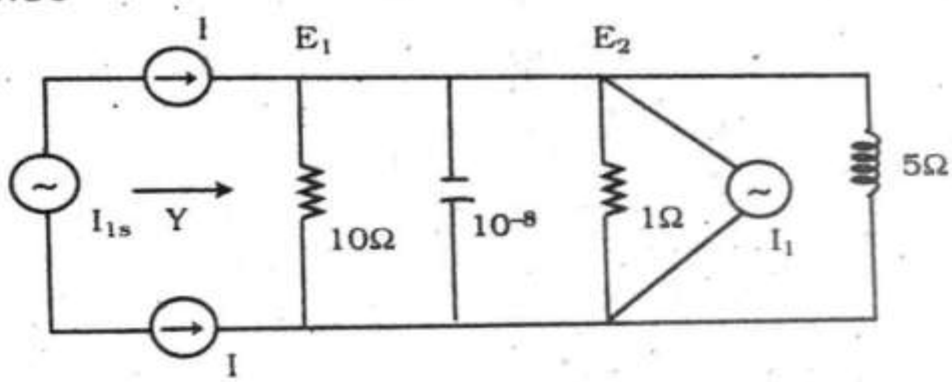
(c) $Q_S = \frac{\omega_m L}{R_S} = \frac{5 \times 10^7 \times 20 \times 10^{-6}}{50} = 20$

5.58

(a) $I = \frac{E}{R_B} = \frac{200}{20000} = 0.01$ A

(b) $V = IR_P = 0.01 \times 10012.5 = 100.125$ V

5.59



$Y = \frac{1}{10} + \omega \times 10^{-6} + \omega \times 10^{-7} + \frac{1}{1} + \frac{1}{5}$

$= 0.1 + \omega \times 10^{-6} + \omega \times 10^{-7} + 1 + 0.2$

$= 1.3 + \omega \times 10^{-6} + \omega \times 10^{-7}$

where,

$I_1 = 0.1\, E_1$

so, $Y_1 = \frac{I_1}{E_1} = 0.1$

$Y_{total} = Y + Y_1$

$= 1.3 + \omega \times 10^{-6} + \omega \times 10^{-7} + 0.1$

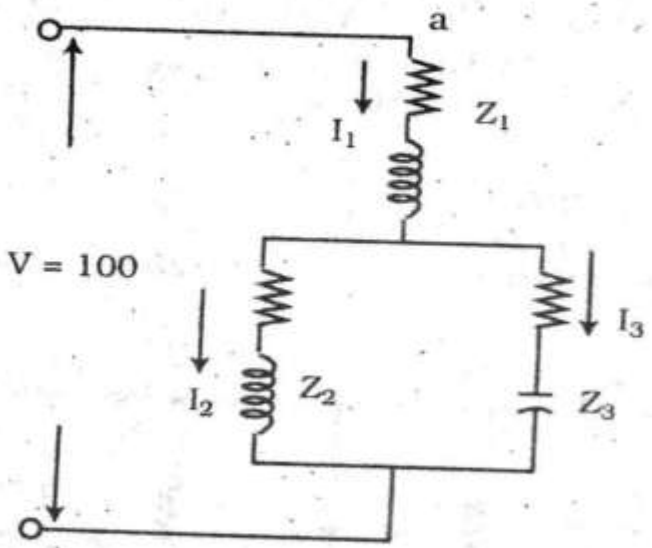
$= 1.4 + \omega \times 10^{-6} + \omega \times 10^{-7}$

where, $\omega = 10^6$ rad/sec

so, $Y_{total} = 1.4 + 10^6 \times 10^{-6} + 10^6 \times 10^{-7}$
$= 1.4 + 1 + 0.1$
$= 2.5$

5.60

$Z_1 = 10 + 30j$
$Z_2 = 5 + 10j$
$Z_3 = 4 - 16j$



$$Z_{bc} = \frac{Z_2 Z_3}{Z_2 + Z_3}$$

$$= \frac{(5 + j10)(4 - j6)}{5 + j10 + 4 - j6}$$

$$= \frac{20 + 160 + j(40 - 80)}{9 - j16}$$

$= 15.89 + j6.15$

$= 17.04\angle 21.16°$

5.61

$Z_{ac} = Z_{bc} + Z_{ab}$
$= 15.89 + j6.15 + 10 + j30$
$= 25.89 + j36.15$
$= 44.46\angle 54.39°$ ohm

(a) $I_1 = \frac{V}{Z_{ac}} = \frac{100}{44.46\angle 54.39} = 2.25\angle -54.4°$ amp

$V_{ab} = I_1 Z_1 = (2.25\angle -54.4)\,(10 + j30)$

$= (2.25\angle -54.4)\,(31.62\angle 71.6)$

$V_1 = 71.15\angle 17.2 = 67.98 + j20.99$

$\therefore \quad V_{bc} = 100 - 67.98 - j20.29$

$= 32.02 - j20.99 = 39.29\angle -33.25$

$I_2 = \frac{38.29\angle -33.25}{5 + j10}$

$= \frac{38.29\angle -33.25}{11.18\angle 63.43} = 3.45\angle -96.7°$ amp

similarly, $I_3 = 2.33\angle 42.7°$

(c) Watts = $VI\cos\theta = 100 \times 2.25 \sin 54.39 = 131$

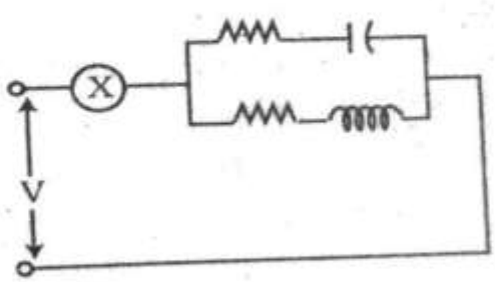
Vars = $VI\sin\theta = 100 \times 2.25 \sin 54.39 = 183$

5.62

$P_1 = I_1^2 R_1 = (2.25)^2\,(10) = 50.625$ watts

$P_2 = I_2^2 R_2 = (3.43)^2\,(5) = 58.8$ watts

$P_3 = I_3^2 R_3 = (2.33)^2\,(4) = 21.7$ watts



$Z_{ab} = \frac{(3 - 4j)\,(1 - j10)}{3 - 4j + 1 + j10} = 9.969\angle -25.12 = 6.31 - j2.96$

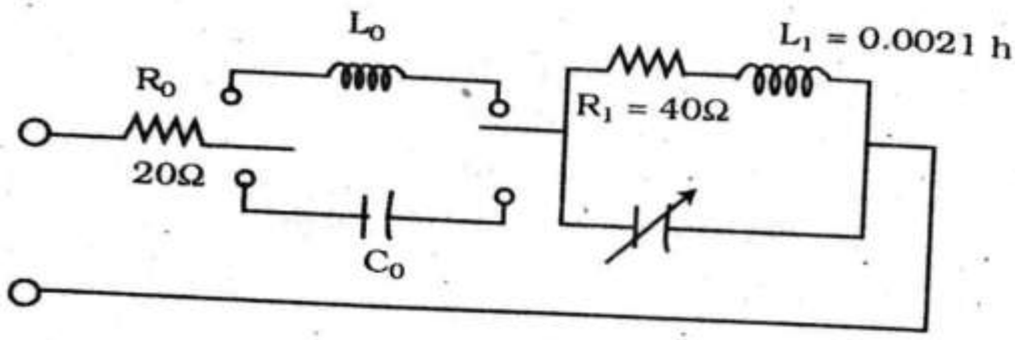
Now, $\tan \cos^{-1} 0707 = \frac{X - 2.96}{6.31}$

or, $\pm 1 = \frac{X - 2.96}{6.31}$

taking $\pm$ve $\quad X = 9.27$ ohm (inductive)

taking $-$ve $\quad X = 3.35$ ohm (capacitive)

5.63



From 15000 Hz – $X_{L1} = 2\pi \times 15000 \times 0.002 = 188.496$ ohm

(a) For parallel resonance $b_L = bc$ or $\frac{188.496}{40^2 + 188.496} = \omega C_L$

$C_2 = 5.39 \times 10^{-8} = 0.0539\ \mu F$

(b) Now,

$$Z_{bc} = \frac{(40+j2\pi\times45000\times0.002)\left(-j\times\frac{1}{2\pi\times45000\times5.39\times10^{-8}}\right)}{40 + j(2\pi\times45000\times0.002 - 5.39 \times 10)}$$

$= 74.23\angle{-89.49^\circ}$ (capacitive)

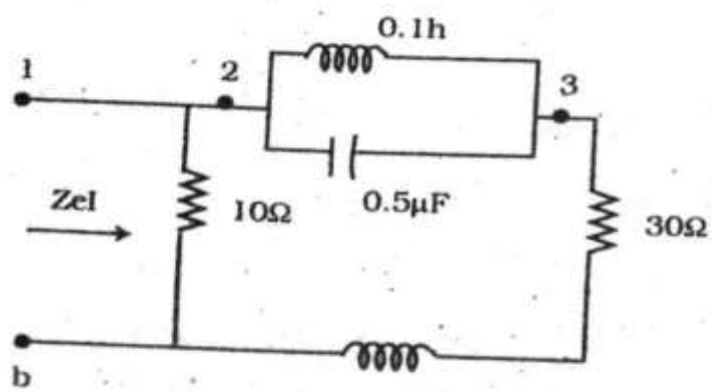
$f = 1592$ Hz

$\therefore \quad \omega = 2\pi f = 10003$ rad/sec

$X_{L1} = \omega L_1 = 1000.3$ ohm

$X_{L2} = \omega L_2 = 500.1$ ohms

$X_C = \frac{1}{\omega C} = 1000$ ohms



$$Z_{12} = \frac{(j1000.3)(-j1000)}{j1000.3 - j1000}$$

$$= \frac{1000300}{j0.583}$$

$$= -j171578$$

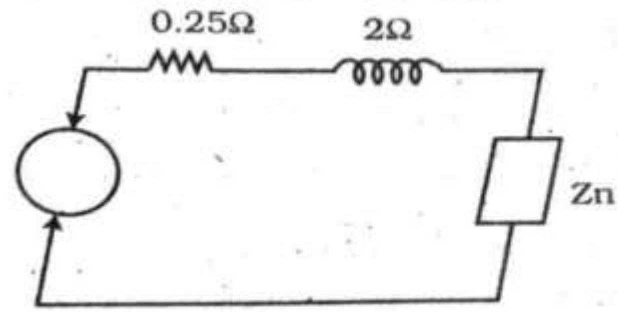
$$Z_{13} = 30 + j500.1 - j1715780 = 30 - j1715280$$

$$Z_{ab} = \frac{(30 - j1715780)(10)}{40 - j1715280}$$

$$= (1715280\angle 90^\circ)(10) = 10\angle 0$$

5.65

$$Z_1 = (0.25 + 0.5) + j(1 + 2) = 0.75 + j3$$



$\therefore$ For max. power $Z_r = 0.75 - j3$

$$I = \frac{20}{0.75 + j3 + 0.75 - j3}$$

$$= \frac{20}{1.5} = 13.33 \text{ A}$$

$\therefore$ $P_{load} = I^2R = 13.33^2 \times 0.75 = 133.33\ \omega$

P_{ine} loss $= I^2 \times 0.25 = 44.44\ \omega$

$P_{loss} = I^2 \times 0.5 = 88.89\ \omega$

5.66

(a) $P_r = I^2R_r = \dfrac{E_g^2R_r}{(R_1 + R_r)^2 + (X_1 + X_r)^2}$

load power would be max^m

$\therefore$ if X_r is zero, since all the terms in the eq^n is constant.

$\therefore$ The condition is $X_r = 0$

(b) $P_r = \dfrac{20^2 \times 0.75}{(0.75 + 0.75)^2 + 3^2} = 26.67\ \omega$

5.67 $P_r = \dfrac{Eg^2R_r}{(R_1 + R_r)^2 + X_1^2}$ – Now, $\dfrac{dP_r}{dR_r} = 0$

or $R_r^2 = X_1^2 = X_1^2 + R_1^2$

Again, for max^m power $R_r = \dfrac{Z_1}{\sqrt{1 + K^2}}$

here,

$$K = \frac{X_r}{R_r} = 0$$

or, $Z_1 = \sqrt{X_1^2 + R_1^2} = \sqrt{0.75^2 + 3^2} = 3.092$ ohm

Now,

$$I = \frac{20.}{0.75 + j\,3 + 3.092} = \frac{20}{4.87\angle 37.98} = 4.1\angle{-37.98^\circ}$$

$P_{load} = |I|^2 \times R_r = 51.98\ \omega$

$P_{ine} = |I|^2 \times 0.25 = 4.2\ \omega$

$P_{gen} = |I^2| \times 0.5 = 8.4\ \omega$

5.68 $K = \dfrac{X_r}{R_r} = 5$ for max^m P_r, we have $R_r = \dfrac{Z_1}{\sqrt{1 + K^2}}$

$$= \frac{0.75^2 + 3^2}{\sqrt{1 + 5^2}} = 0.606 \text{ ohm}$$

$$= 5R_5 = 3.03 \text{ ohm}$$

$$Z_{load} = 0.606 - j\,3.03$$

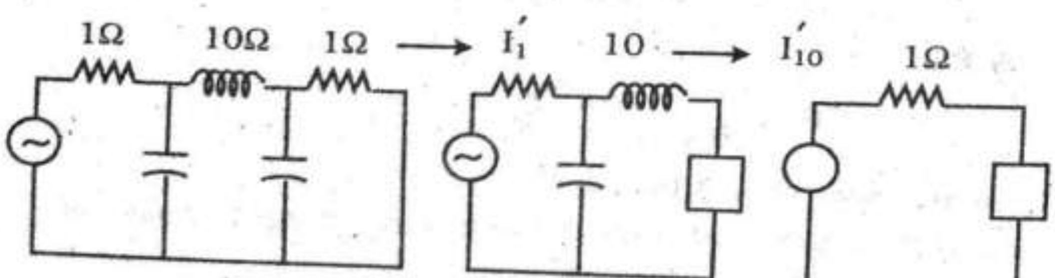
$$I = \frac{20}{(0.75 + 0.606)^2 + (3 - 3.03)^2}$$

$$= 14.74 \text{ A}$$

$\therefore \quad P_{max} = (14.74)^2 \times 0.606$

$$= 131.76$$

5.69



$$Z_P = \frac{1\angle 0 \times 5\angle{-90}}{1 + j0 + 0 - j5} = \frac{5\angle{-90}}{1 - j5} = \frac{5\angle{-90^\circ}}{5.9\angle{-78.69}}$$

$$= 0.98\angle{-11.31}$$

$$Z_1 = 0 + j10 + 0.96 - j0.192 = 0.96 + j9.8 = 9.85\angle 84.1$$

$$Z_P = \frac{5 \times 1\angle{-90} \times 9.85\angle 84.4}{0.96 + j9.8 - j5}$$

$$= \frac{49.25\angle{-5.6}}{0.96 + j4.8} = \frac{49.25\angle{-5.6}}{4.9\angle 76.69}$$

$$= 10\angle{-82.29} = 1.34 - j9.91$$

$$Z_{eq} = 1 + 1.34 - j9.91 = 2.34 - j9.91 = 10.18\angle{-76.71}$$

$$I_1' = \frac{100\angle 0}{10.18\angle{-76.71}} = 9.82\angle 76.71$$

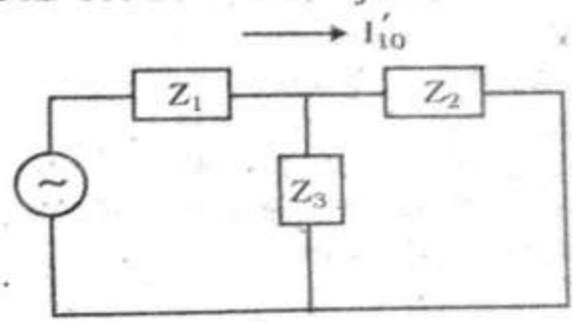
$$I_{10'} = I_1' \times \frac{5\angle{-90°}}{0 - j5 + 0.96 + j9.8} = \frac{9.82\angle 76.71 \times 5\angle{-90}}{4.90\angle 78.69}$$

$$I_1' = I_1 \times \frac{5\angle{-90}}{0.96 + j4.8} = 10.02\angle{-91.98} = 0.35 - j10$$

$$Z_2' = I_{10}' \times \frac{Z_3}{Z_2 + Z_3} = I_{10} \times \frac{5\angle{-90}}{1 + j0 + 0 - j5}$$

$$= \frac{10.02\angle{-91.98} \times 5\angle{-90°}}{5.09\angle{-78.69}}$$

$$= 9.84\angle{-103.29} = -2.26 - j9.58$$



when,

$$F_1 = 0,\ I_2 = \frac{50\angle 60}{10.18\angle{-76.71}} = 4.91\angle 138.71 = 3.57 + j3.37$$

$$I_2 = I_2 + I_2' = -2.26 - j9.58 - 3.57 + 3.37j$$

$$= 8.52\angle{-133.19°}$$

5.70

$Z_1 = 10\ j30$ ohms

$Z_2 = 5 + j10$ ohms.

$Z_3 = 4 - j16$ ohms

If $100\angle 0$ volts remains at the branch 1. calculation of the currents are as follows :

$$Z_{ckt} = Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3} = (10 + j30) + \frac{(5 + j10)(4 - j16)}{5 + j10 + 4 - j16}$$

$$= 10 + j10 + \frac{20 + 160 + j(48 - 8)}{9 - j16}$$

$$= 10 + j30 + \frac{1620 + 240 + j(1080 - 360)}{81 + 36}$$

$$= 44.47 \angle 54.38° \text{ ohms}$$

$$\therefore \quad I = \frac{100}{44.47\angle 54.38}$$

$$= 2.25\angle 54.38° \text{ amps}$$

$$I_1 Z_1 = (2.25\angle 54.38°)(31.62\angle 71.54°)$$

$$= 71.145\angle 17.185$$

$$= 67.969 + j21$$

$$V_2 = 100 + j0 - 67.969 - j21$$

$$= 32 - j21$$

$$= 38.3\angle -33.25$$

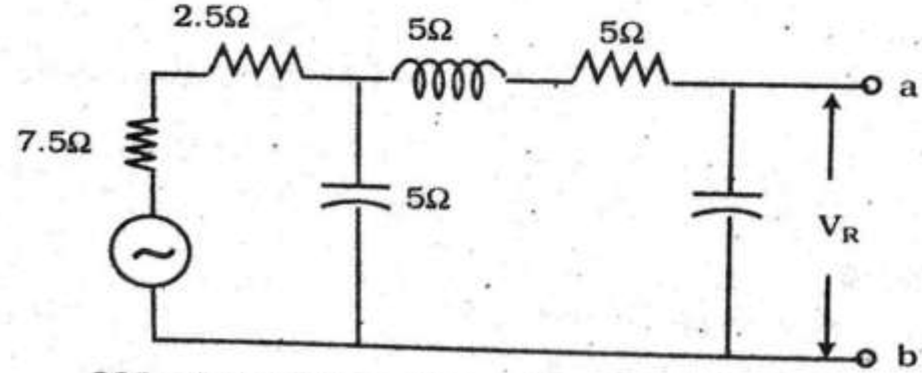
$$I_3 = \frac{38.3\angle 33.25°}{4 - j16}$$

$$= 2.32\angle 42.7° \text{ amp}$$

Now, if the $100\angle 0$ volts is inserted in branch 3, the resulting current in branch 1 would be equal to I_3, according to the reciprocity theorem. Therefore, in this case

$$I_1 = 2.32\angle 42.7° \text{ amp}$$

5.71



$$200 = \{7.5 + j(2.5 - 5)\} I_1 + j5I_2$$

$0 = j5I_1 + \{5 + j(5 - 5 - 5)\}I_2$

or. $I_1 = \frac{5-5j}{-j5} I_2$

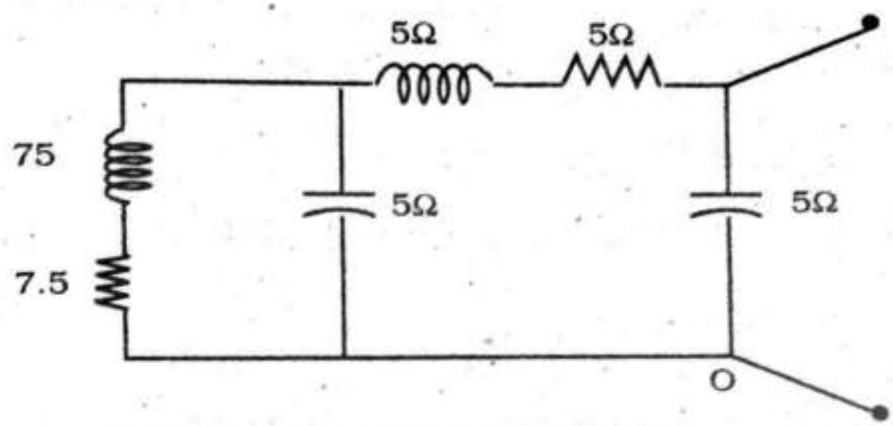
$= \left(-\frac{1}{j} + 1\right) I_2 = (1 + j)\, I_2$

Now. $200 = (7.5 - 2.5j)\,(1 + j)\,I_2 + j5I_2 = (10 + 5j + 5j)\,I_2$

or. $I_2 = \frac{200}{10 + j10} = 14.14\angle{-45^\circ}$

$V_R = (-5j) \times I_2 = (5\angle{-90^\circ})\,(14.14\angle{-45^\circ}) = 70.7\angle 135 = 50 - j50$

Now,

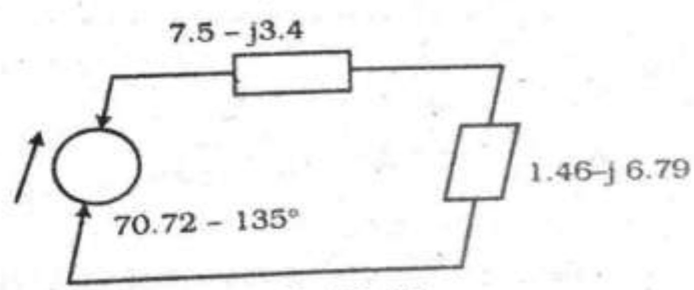


$$Z = \frac{(8.06\angle 7.13)\,(5\angle{-90})}{8 + j1 - j5} = \frac{40.30\angle 82.87}{8.94\angle{-26.57}} = 4.5\angle{-56.3^\circ}$$

$\therefore \quad I_{ab} = \frac{70.7\angle{-135^\circ}}{2.5 + 1.46 + j\,(6.78 - 3.74)}$

$\doteq \frac{70.7\angle{-135}}{5\angle 37.5} = 14.14\angle{-172}$

5.72



$Z_1 = 1 + j1 + 2 + j3 = 5\angle 53.13$

$Z_2 = 1 + j3 + 2 + j1 = 3 + j4 = 5\angle 53.13$

$$Z_3 = -j5 = 5\angle -90°$$

$$Z = \frac{Z_1 Z_3}{Z_1 + Z_3}$$

$$= \frac{5\angle 53.13 \times 5\angle -90}{3 + j4 - j5}$$

$$= \frac{252\angle -36.87}{3 - j1}$$

$$= \frac{25\angle -36.87}{3.16\angle -18.43}$$

$$= 7.91\angle -18.44$$

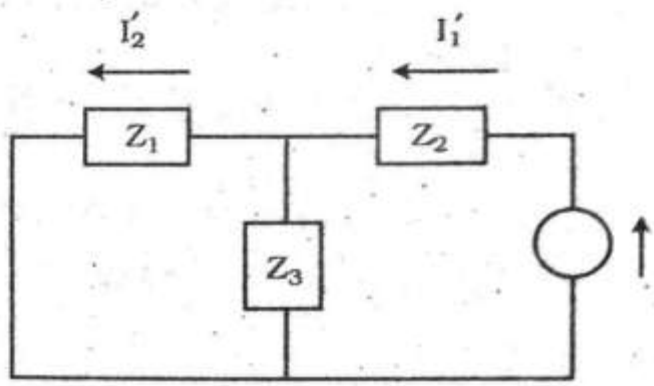
$$Z = 7.5 - j2.5 + 3 + 4j = 10.5 + j1.5 = 10.61\angle 8.13$$

$$I_{1'} = \frac{100\angle 0}{10.61\angle 8.13} = 9.43\angle -8.13$$

$$Z = \frac{Z_2 \times Z_3}{Z_2 + Z_3}$$

$$= \frac{5\angle 53.13 \times 5\angle -90}{3 + j4 - j5} = 7.5 - j2.5$$

$$I_2'' = \frac{50\angle 26.59}{10.61\angle 8.13} = 4.71\angle 18.46$$



$$I_1'' = I_2'' \times \frac{Z_3}{Z_2 + Z_3}$$

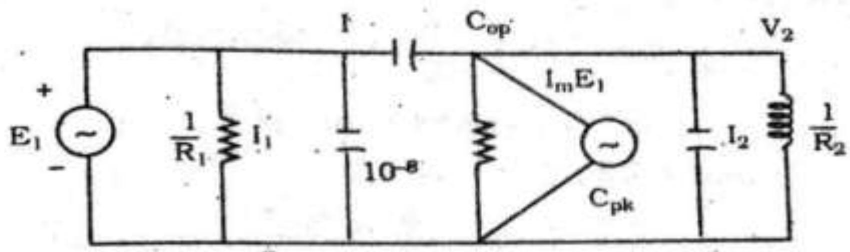
$$= 4.71\angle 18.46 \times \frac{5\angle -90°}{3.16\angle 18.43}$$

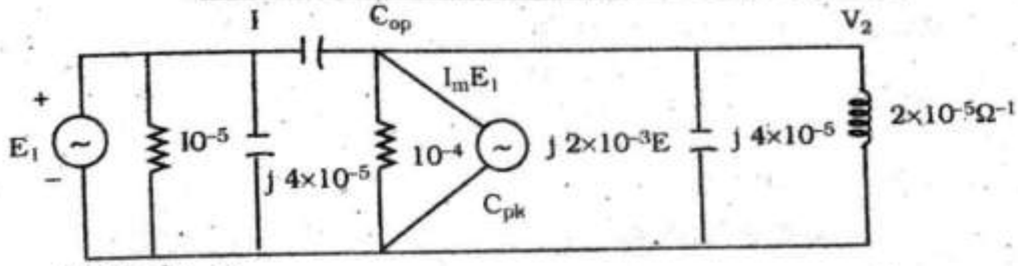
$$= 7.45\angle -53.11$$

$$\therefore \quad I_1 = I_1'' - I_1' = 4.47 - 5.96j - 9.33 + j1.33$$

$$= 6.71\angle -136.39$$

5.73





at Node g_1.

$$V_{g1} = E_1$$

at Node g_2.

$$V_{g2} \times (10^{-4} + j4 \times 10^{-5} + 2 \times 10^{-5}) - v_{g1}\,(j0.5 \times 10^{-5})$$

$$= 2 \times 10^{-3}\,E$$

$$V_{g2}\,(10^{-4} + j4 \times 10^{-5} + 2 \times 10^{-5} + 0.5 \times 10^{-5}) - E_1\,(j0.5 \times 10^{-5})$$

$$= 2 \times 10^{-3}\,E_1 \quad [\text{Replacing } V_{g1} = E_1\,]$$

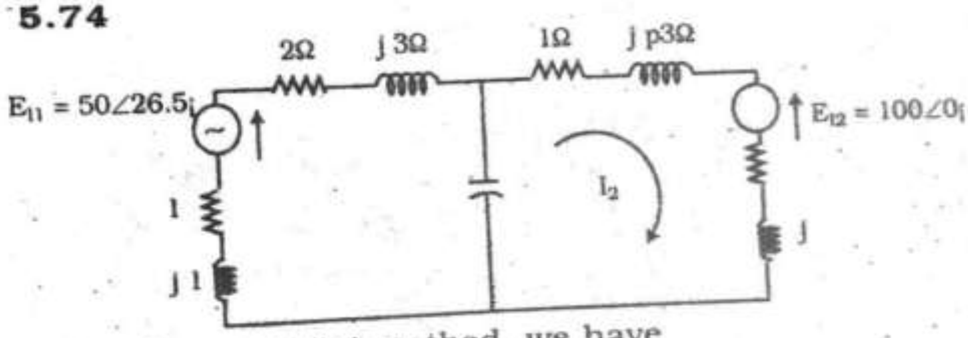
$$V_{g2}\,(10^{-4} + j4 \times 10^{-5} + 2 \times 10^{-5} + 0.5 \times 10^{-5})$$

$$= E_1\,(j0.5 \times 10^{-5} + 2 \times 10^{-3})$$

$$V_{g2} = \frac{1\angle 0^\circ\,(j0.5 \times 10^{-5} + 2 \times 10^{-3})}{(10^{-4} + j4 \times 10^{-5} + 2 \times 10^{-5} + 0.5 \times 10^{-5})}$$

$$= 15.25\angle{-17.60}$$

5.74



Using loop current method, we have

$$I_A = 6.72\angle{-136.39^\circ}$$

$$\therefore \quad V_{ab} = (6.72\angle{-136.39^\circ})\,(1\angle 0) = 6.72\angle{-136.39^\circ}$$

Now,

$$Z_{PQ} = \frac{(3 + j4)(5\angle{-90^\circ})}{3 + 4j - 5j} = \frac{25\angle{-36.87}}{3.16\angle{-18.43}}$$

$$= 7.91\angle{-18.44} = 7.5 - j2.5$$

$$Z_{PQR} = 7.5 - j2.5 + j(1 + 3)$$

$$= 9.5 + j1.5 = 9.62\angle 8.97^\circ$$

Which is parallal to 10 hm

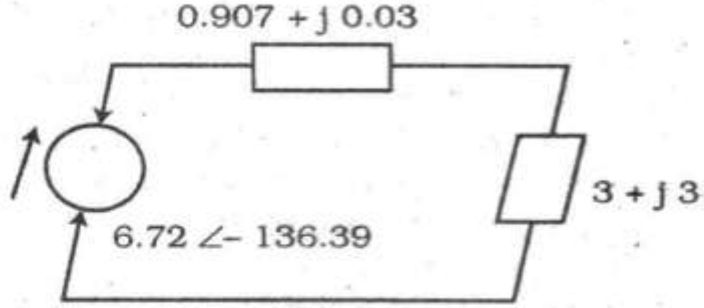
$$\therefore \quad Z = \frac{(9.62\angle 8.97)(1\angle 0)}{9.6 + j1.571}$$

$$= \frac{9.62\angle 8.97}{10.6\angle 8.11}$$

$$= 0.9.7\angle 0.81$$

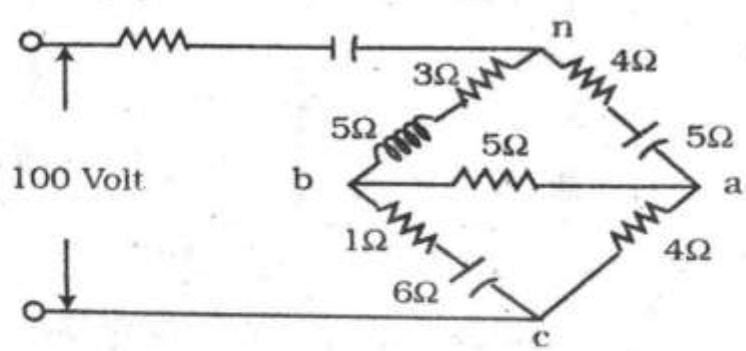
$$= 0.907 + j0.013$$

So the thevenin's equivalent ckt is



$$\therefore \quad I = \frac{6.72\angle 136.39}{3.907 + j3.013} = 1.36\angle{-174}$$

5.75



$$Z_{ca} = \frac{(5 - j30)\,(10 + j6)}{136} = \frac{50 + 180 - j300 + j30}{136}$$

$$= 1.69 - j1.99$$

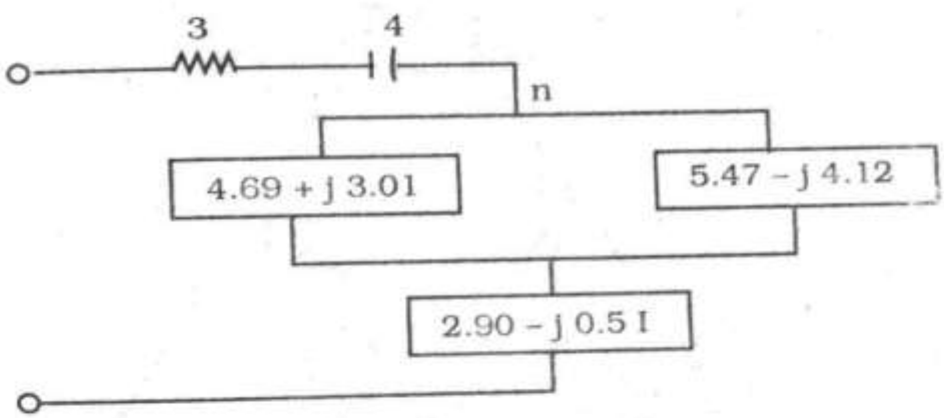
$$Z_{ba} = \frac{(1 - j6)\,(4)}{10 - j6} = \frac{(4 - j24)\,(10 + j6)}{136}$$

$$= 2.09 - j1.59$$

$$Z_{bc} = \frac{20}{10 - j6} = 1.47 + j0.88$$

$$Z_{nbd} = 3 + j5 + 1.69 - j1.96 = 4.69 + j3.01$$

$$Z_{nad} = 5j + 1.47 + j0.88 = 5.47 - j4.12$$



$$Z_{nac} = \frac{(4.69 + j3.02)\,(5.47 - j4.12)}{4.69 + j3.01 + 5.47 - j4.12}$$

$$= \frac{(5.573\angle 32.69)\,(6.85\angle -36.90)}{10.22\angle -6.23}$$

$$= 3.735\angle 1.93 = 3.73 + j0.126$$

$$Z_{nc} = 3.73 + j0.126 + 2.09 - j1.59$$

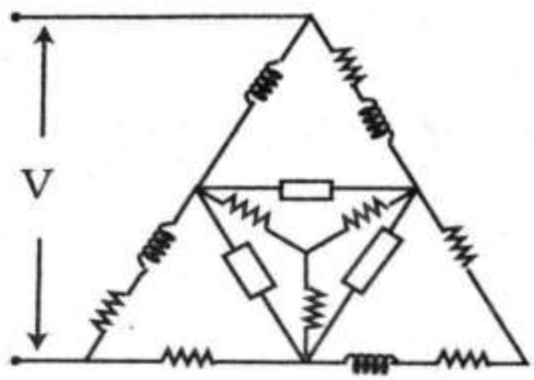
$$= 5.82 - j1.454$$

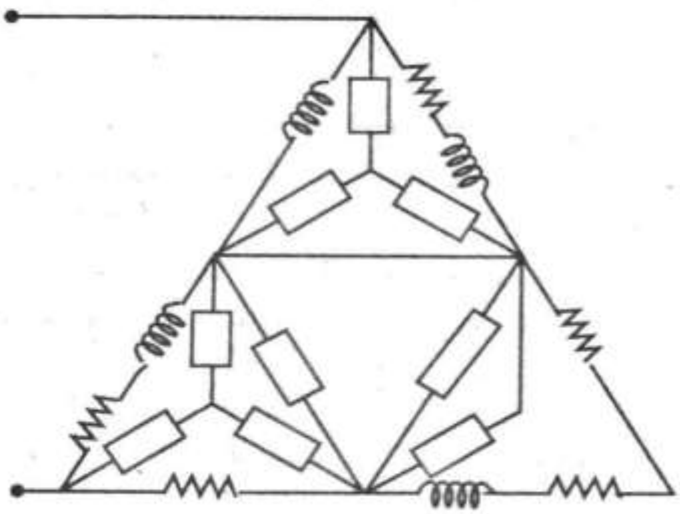
$$\therefore \quad Z_{Ckt} = 3 - j4 + 5.82 - j1.464$$

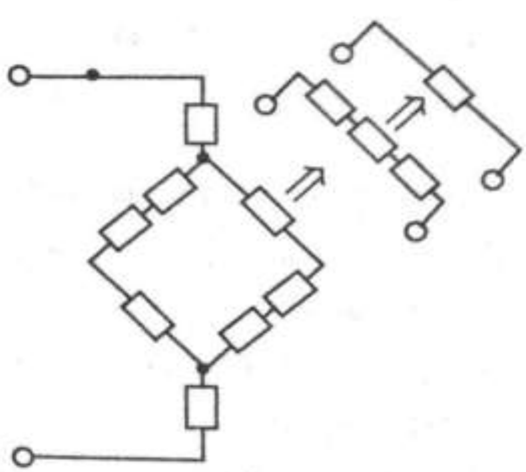
$$= 8.82 - j5.464$$

$$= 10.375\ \angle 31.78^\circ$$

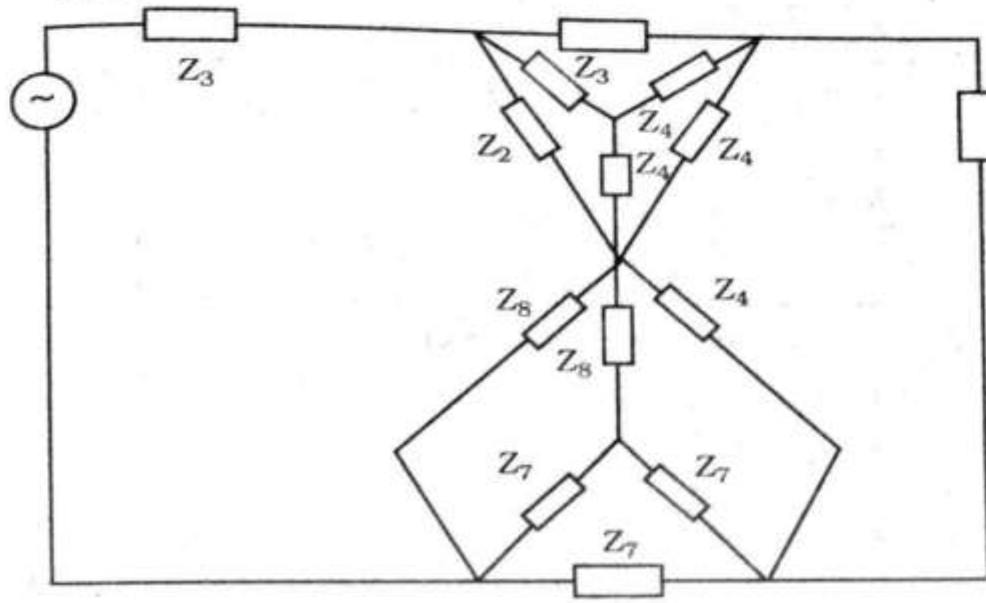
5.78



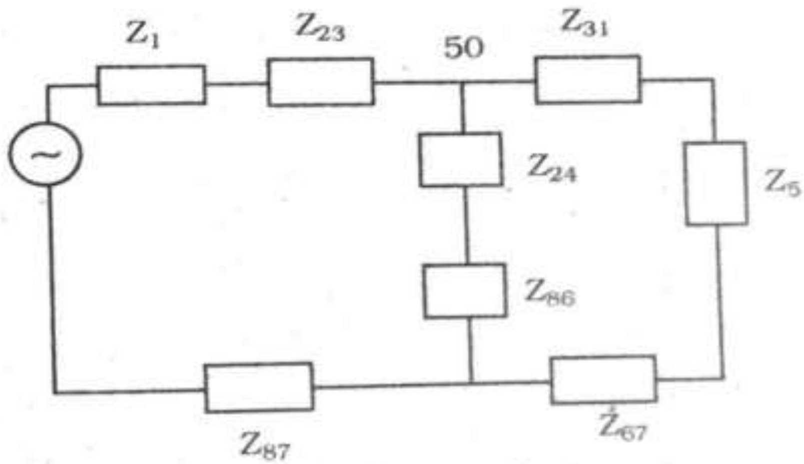




5.79



The current $I_3 = 0$ i.e. the current through Z_5 is zero, if $Z_{15} = 0$ i.e. if the points P and Q are short circuited.



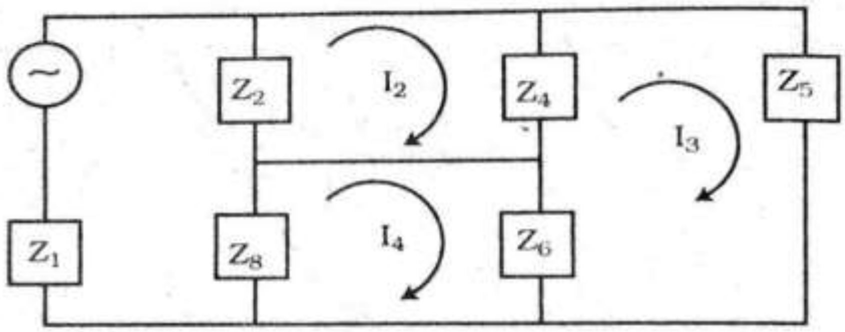
Hence,

$$Z_{PQ} = 0$$

or, $Z_{24} + Z_{68} = 0$

or, $$\frac{Z_2 Z_4}{Z_2 + Z_3 + Z_4} + \frac{Z_6 Z_8}{Z_6 + Z_7 + Z_8} = 0$$

or, $Z_2 Z_4 (Z_6 + Z_7 + Z_8) + Z_6 Z_8 (Z_2 + Z_3 + Z_4) = 0$



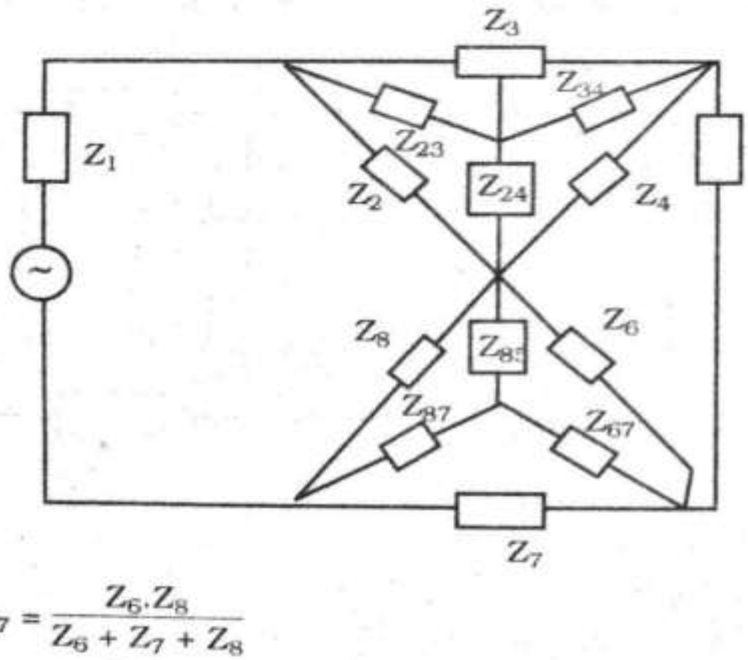
Z_1, Z_3, Z_4 are in delta connection

When transformed to star,

$$Z_{23} = \frac{Z_2 Z_3}{Z_2 + Z_3 + Z_4}$$

$$Z_{34} = \frac{Z_3 Z_4}{Z_2 + Z_3 + Z_4}$$

$$Z_{24} = \frac{Z_2 Z_4}{Z_2 + Z_3 + Z_4}$$



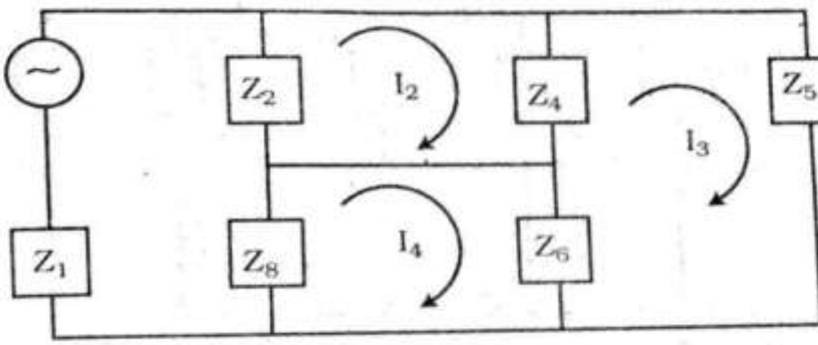
$$Z_{67} = \frac{Z_6 . Z_8}{Z_6 + Z_7 + Z_8}$$

$$Z_{78} = \frac{Z_7 Z_8}{Z_7 + Z_8 + Z_5}$$

$$Z_{86} = \frac{Z_8 Z_6}{Z_8 + Z_6 + Z_7}$$

What relation relationship between the Z's of fig. will make $I_3 = 0$ regardless of the magnitude of E_{in}.

Z_2, Z_3, Z_4 are in delta connection



$$Z_{23} = \frac{Z_2Z_3}{Z_2+Z_3+Z_4},\quad Z_{34} = \frac{Z_3Z_4}{Z_2+Z_3+Z_4},\quad Z_{24} = \frac{Z_2Z_4}{Z_2+Z_3+Z_4}$$

Z_6, Z_7, Z_8 are transformed to star connection.

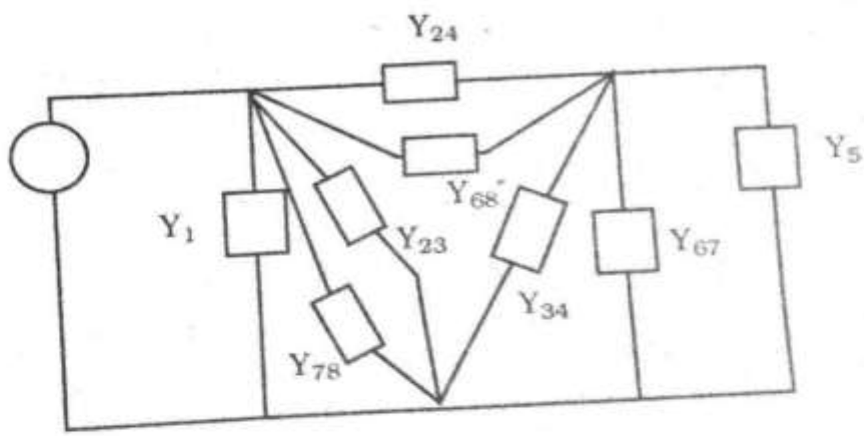
$$Z_{68} = \frac{Z_6Z_8}{Z_6+Z_7+Z_8},\quad Z_{78} = \frac{Z_7.Z_8}{Z_6+Z_7+Z_8},\quad Z_{61} = \frac{Z_6Z_7}{Z_6Z_7+Z_8}$$

Now, when $Z_{24} + Z_{68} = 0$ then $I_3 = 0$ of any magnitute of E_{in}.

The relationshiop is $Z_{24} + Z_{68}$

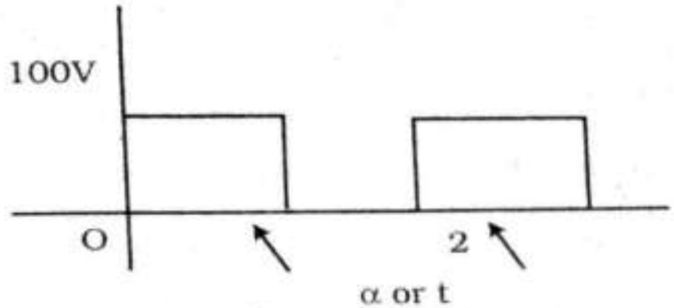
$$= \frac{Z_2Z_4}{Z_2+Z_3+Z_4} + \frac{Z_6Z_8}{Z_6+Z_7+Z_8} = 0$$

5.80



Chapter–6
Non-Sinusoidal Waves

6.1



$$C_n = \frac{1}{\pi}\int_0^{\pi} 100\, e^{-jnt}\, dt \qquad \Big|\qquad \omega_n = 2\frac{\pi}{2\pi} = 1$$

$$= \frac{1}{\pi}\int_0^{\pi} 100\, e^{-jnt}\, dt$$

$$= \frac{1}{-j\pi n}\, 100 \times e^{-jnt}\Big|_0^{\pi}$$

$$= \frac{1}{-jn\pi} \times 100 \times [e^{-jn\pi} - 1]$$

$$= \frac{j100}{n\pi}[e^{-jn\pi} - 1]$$

$$e = \sum_{n=-\infty}^{\infty} {}_n e^{jn\omega t}$$

$$= \sum_{n=-\infty}^{\infty} \frac{j100}{n\pi}[e^{-jn\pi} - 1]$$

Problem 2. Do yourself.

6.2 Problem-3 Do yourself

6.3 Poblem-4

$i = 85.50 \sin(\omega t - 15^\circ) + 26.2 \sin(3\,\omega t + 92) + 6.5 \sin(5\,\omega t + 214.2) + 2.39 \sin(7\omega t - 32.7)$

$= -85.50 \cos(\omega t + 75) - 26.2 \cos(3\omega t + 92 + 90) - 6.5 \cos(5\omega t + 214.2 + 90^\circ) - 2.39 \sin(7\omega t - 32.7 + 90^\circ$.

$= -85.50 \cos(\omega t + 75) - 26.2 \cos(3\omega t + 182)$

$- 6.5 \cos(5\omega t + 304.2) - 2.39 \sin(7\omega t + 57.3)$

6.5 $i' = 10 \sin(\omega t + 30°) + 2 \sin 7\omega t$

and, $i'' = 35 \sin(\omega t - 10) + 7 \sin(7\omega t + 80°)$

$\therefore \frac{di'}{dt} = 10 \cos(\omega t + 30) + 2 \times 7 \cos 7\omega t$

$\frac{di''}{dt} = 35 \cos(\omega t - 10) + 7 \times 7 \cos (7\omega t + 80°)$

So $\frac{di'}{dt}$ = variation of i'

$\frac{di''}{dt}$ = variation of i''

and $\frac{di''}{dt} = 35 \cos(\omega t - 10) + 49 \cos(7\omega t + 80°)$

$= 35 \cos(\omega t - 30° + 360) + 49 \cos(7\omega t + 80° + 360°)$

Here, $\frac{10}{35} = \frac{14}{47} = \frac{2}{7}$

phase shift should be = 20°

and at 7th harmonics = 80°

Phase differ is not acceptable.

$\therefore$ they are not alike.

6.6

$$V = V_{m1} \sin(\omega t + \alpha_1) + V_{m3} \sin(3\omega t + 30°)$$

$$V_{eff} = \sqrt{f(V_{m_1}^2 + V_{m_3}^2 . 2)}$$

$$V_{eff} = \sqrt{\frac{1}{T} \int V^2 dt}$$

$$= \sqrt{\frac{1}{T} \int [v_{m1} \sin(\omega t + \alpha_1) + v_{m3} \sin(3\omega t + 30°)]^2 dt}$$

$$= \sqrt{\frac{1}{T} \int [V_{m_1}^2 \sin^2(\omega t + \alpha_1) + 2V_{m1} V_{m3} \sin(\omega t + \alpha_1) \sin(3\omega t + 30°) + V_{m_3}^2 \sin^2(\omega t + 30°)] dt}$$

$$= \sqrt{\frac{1}{T} [V_{m_1}^2 \int \sin^2 (\omega t + \alpha_1) + V_m V_{m3} \int 2 \sin(\omega t + \alpha_1) \sin(3\omega t + 30°) + V_{m_3}^2 \int \sin^2 (\omega t + 30°)] dt}$$

expand $\sin(\omega t + \alpha_1) \sin(3\omega t + 30°)$ as $\cos(A - B) - \cos(A + B)$

Then use the T(period) for one defined frequency as ω, 2ω, 4ω and solve the integration as simple.

6.7 Problem

$e = 100 \sin \omega t + 50 \sin (5\omega t - 80°) - 40 \cos (7\omega t + 30°)$ Volt J

$i = 30 \sin(\omega t + 60°) + 20 \sin(5\omega t - 50°) + 10 \sin(7\omega t + 60°)$

So.

$$P = vi \cos \theta$$

$$= \frac{100 \times 30}{2} \cos (0 - 60) + \frac{50 \times 20}{2} \cos (-80 + 50) + \frac{(-40)(10)}{2} \cos (30 - 60°)$$

$$= 750 + 433.0127 - 173.205$$

$$= 1009.8077 \text{ watt J}$$

6.8 Problem

Effective voltage $= \sqrt{\frac{100^2 + 50^2 + 40^2}{2}} = 83.964$ Volt

Effective current $= \sqrt{\frac{30^2 + 20^2 + 10^2}{2}} = 26.4575$ Volt

Power from the previous problem P = 1009.8077

VA from the previous problem

$= 100 \times 20 + 50 \times 26 + (-40) \times 10 = 3600$ Watt J

Power factor $= \frac{1009.8077}{3600} = 0.28$

$\theta = 73.709$

So $v_{eq} = \sqrt{2} \times 83.964 \sin \omega t$ Volts

$i_{eq} = \sqrt{2} \times 26.4575 \sin(\omega t + 73.709)$

6.9 (a) Here

$$c_3 = \frac{1}{2\pi}\left[\int_0^{\pi} 10e^{-j3t}dt + \int_{\pi}^{2\pi} (-5)\, e^{-3t}dt\right]$$

$$= \frac{1}{2\pi} \times \frac{10}{-j3} [e^{-j3\pi} - 1] + \frac{(-5)}{-3j \times 2\pi} \times [e^{-6\pi j} - e^{j3\pi}]$$

$$= \frac{1}{2\pi} \times \frac{10}{-j3} [e^{-j3\pi} - 1] + \frac{(-5)}{-3j \times 2\pi} \times [e^{-6\pi j} - e^{j3\pi}]$$

$$= \frac{5}{-j3\pi} [e^{-j3\pi} - 1] + \frac{(-5)}{2\pi \times -3j} \times [e^{-6\pi j} - e^{j3\pi}]$$

So, $a_3 = \frac{c_3}{2}$ where $e^{-3\pi j} = \cos(-3\pi) + j \sin(-3\pi)$ and so on.

(b)

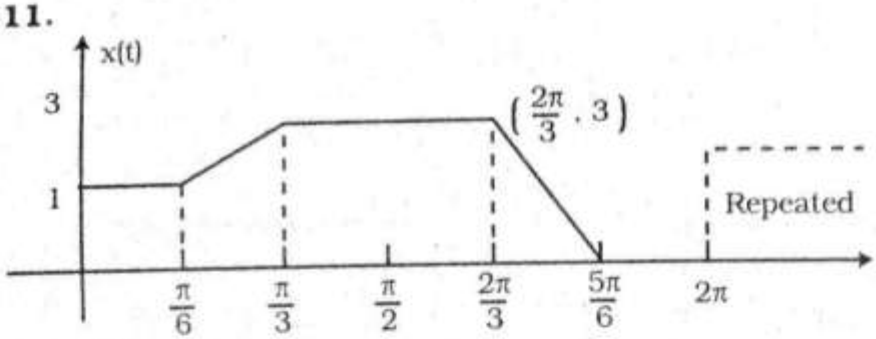
$$= \sum_{n=-\infty}^{\infty} c_n e^{jn\omega 0t}$$

$$= \sum_{n=-\infty}^{\infty} \left[\frac{5}{-j3\pi}(e^{-j3\pi} - 1) + \left(\frac{-5}{2\pi}\right) \times \frac{1}{-3j}(e^{-6\pi j} - e^{j3\pi})\right] \times e^{jnt}$$

(c) Do yourself

6.10 (a) Same to number 9.

6.11.



$$c_n = \frac{1}{2\pi}\int_0^{2\pi} x(t)e^{-jnt}dt$$

$$= \frac{1}{2\pi}[\int_0^{\pi/6} e^{-jnt}\,dt + \int_{\pi/6}^{\pi/3}\left(\frac{2}{\pi/3}t\right) e^{-jnt}\,dt + \int_{\pi/3}^{2\pi/3} 3e^{-jnt}\,dt$$

$$+ \int_{2\pi/3}^{5\pi/6} [\frac{18}{\pi}(t - 2\pi/3) + 3]dt$$

As these integrations are easier to do and can be evaluated by by parts formula, try yourself.

6.12 Not important

6.13 Not important

6.14

$$V = 4\sin\omega t - 3\cos\omega t - 7.66\sin 2\omega t + 6.43\cos 2\omega t$$
$$- 2\sin 3\omega t - 1.5\cos 3\omega t$$
$$= \sqrt{4^2 + 3^2}\sin(\omega t - \tan^{-1} 4/3) - \sqrt{(7.66)^2 + (6.43)^2}$$
$$\sin(2\omega t - \tan^{-1}\frac{7.66}{6.43}) - \sqrt{2^2 + 1.5^2}\sin(3\omega t + \tan^{-1} 2/1.5)$$
$$= 5\sin(\omega t - 53.13) - 10\sin(2\omega t - 49.98)$$
$$- 2.5\sin(3\omega t + 53.13)$$

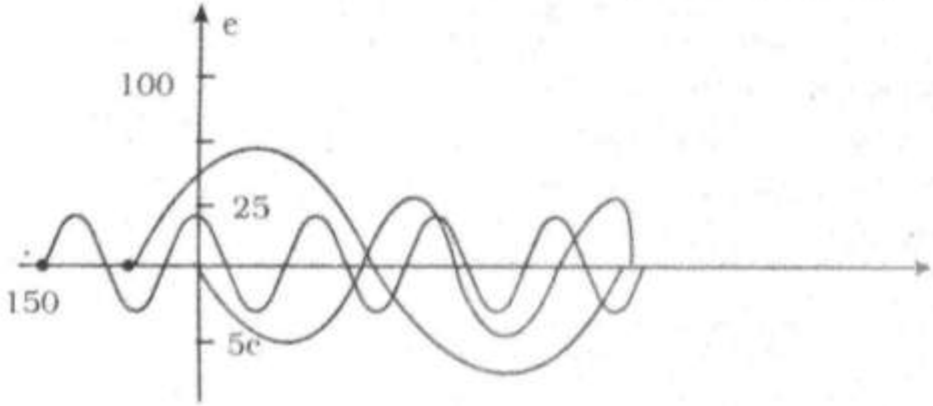
6.15 Not important

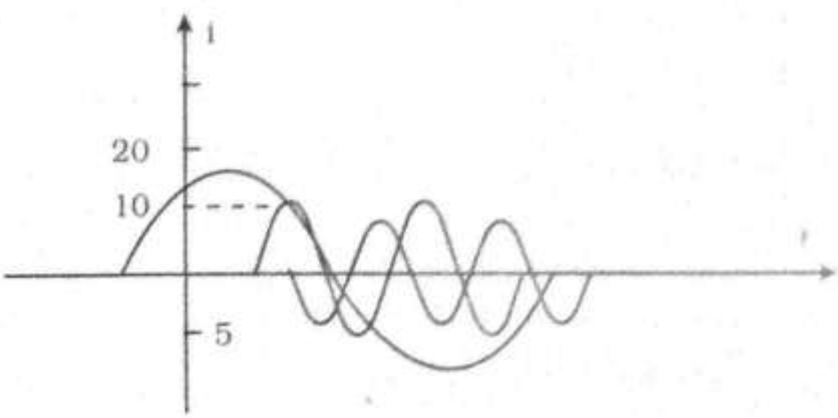
6.16 Not important

6.17 Not important

6.18 $e = 100 \sin(\omega t + 30) - 50 \cos 2\omega t + 25 \sin(5\omega t + 150^\circ)$

and, $i = 20 \sin(\omega t + 40) + 10 \sin(2\omega t + 30) - 5 \sin(5\omega t - 50)$





ratio $= \frac{100}{20} = \frac{50}{10} = \frac{25}{5}$. phone difference = 10°

So it is symmetric.

6.19 Similar to 18

6.20

$v = 100 \sin(\omega t + 70) - 60 \sin(2\omega t - 30^\circ) + 30 \sin(3\omega t - 60^\circ)$

$i = 50 \cos(\omega t - 60^\circ) + 30 \sin(2\omega t + 70) - 15 \cos(3\omega t - 90^\circ)$

From $v = 100\sin(\omega t - 60 + 90) + 60\sin(2\omega t + 70 + 180^\circ)$
$+ 30\sin(3\omega t - 90 + 3\times 90^\circ)$
$= 100\sin(\omega t + 30) + 60\sin(2\omega t + 250) + 30(3\omega t + 180^\circ)$

ratio $= \frac{100}{100} = \frac{60}{60} = \frac{30}{60} = 1$ and phase shift 30°

6.21 $e = 100\sin(\omega t - 20) + 50\sin(3\omega t + 60) - 25\cos(5\omega t - 30^\circ)$

and, $i = 20\cos(\omega t - 60^\circ) - 10\sin(3\omega t + 15^\circ) + 5\sin(5\omega t - 70^\circ)$

ratio $r = \frac{100}{20} = \frac{50}{10} = \frac{25}{5} = 5$

Phase shift = 10°

at 3rd harmonics = 30°

and at 5th harmonics = 50°

So they are not alike.

6.22

$$V_{eff} = \sqrt{\frac{100^2 + 50^2 + 25^2}{2}}$$
$$= 81.0093 \text{ volt}$$
$$i_{eff} = \sqrt{\frac{20^2 + 10^2 + 5^2}{2}}$$
$$= 22.9128 \text{ amp}$$

6.23

$$V = 100\sin(\omega t + 30^\circ) - 40\sin(2\omega t - 30^\circ)$$
$$+ 40\sin(2\omega t + 30^\circ) + 20\cos(5\omega t - 30^\circ)$$
$$\therefore \quad V_{eff} = \sqrt{\frac{100^2 + 40^2 + 40^2 + 20^2}{2}}$$
$$= \sqrt{\frac{13600}{2}}$$
$$= 82.462$$

6.24 The voltmeter value of the complex wave,

$$V = \sqrt{\frac{100^2 + 70^2 + 50^2}{2}}$$
$$= \sqrt{8700}$$
$$= 93.2737 \text{ volt}$$

6.25

$$i = 2.5 + \frac{30}{\pi}\sin x + \frac{30}{3\pi}\sin 3x + \frac{30}{5\pi}\sin 5x + \frac{30}{7\pi}\sin 7x + \frac{30}{9\pi}\sin 9x + \ldots\ldots\ldots$$

or, $I^2_{m(ae)} = 100 - 16$

or, $I_m(ac) = \sqrt{84} = 9.165$

6.26 in sketch yourself

(a) $i = 5 + 2\sin(754\ t)$

$$\frac{di}{dt} = 2 \times 754\cos(754\ t)$$

$$= 1508\cos(754\ t)$$

$\therefore\ \cos(754\ t) = 0$

$754\ t = 1$

$$t = \frac{1}{754}$$

(b) $I_{m(ac)} = 0.5(7 - 1)$

$= 3$ volt

(c) $$V_{eff} = \sqrt{\frac{5^2 + 2^2}{2}}$$

$= 3.81$ volt

6.27 (a) $i = I_{dc} + I_{m(ac)}\sin x$

yes, $I_{m(ac)}$ is the maximum value.

(b) $i = I_{dc} + I_{m(ac)}\sin x$

So $I_c = 4$ and

$$i_{eff} = \sqrt{\frac{I_{dc}^2 + I_m^2(ac)}{2}}$$

$$4 \times 25 = 16 + I_{m\ (ac)}^2$$

$$\frac{I_{m2}}{I_{m1}} \times 100 = \frac{\frac{(I_{max} + I_{min}) - 2I_0}{4}}{(I_{max} - I_{min})/2}$$

$$= \frac{0.5(I_{max} - I_{min}) - I_b}{(I_{max} - I_{min})}$$

(c) Percent distortion = $(I_{m2}/I_{m1}) \times 100$

$I_{m1} = 0.5(I_{max} - I_{min})$

or, $2I_{m1} = (I_{max} - I_{min})$

and $4I_{m2} = (I_{max} + I_{min}) - 2I_b$

$I_{max} + I_{min} = 4I_{m2} + I_b$

and $I_{max} - I_{min} = 2I_{m1}$

$2I_{max} = (4I_{m2} + 2I_b + 2I_{m1})$

$I_{max} = (2I_2 + I_b + I_{m1})$

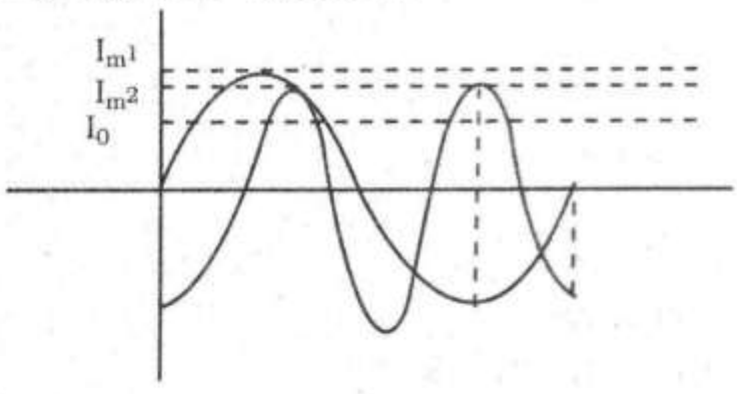
and $2I_{min} = 4I_{m2} + 4I_b - 2I_{m1}$

$I_{min} = (2I_{m2} + I_b - I_{m1})$

So find the ratio and I_{max}, I_{min} from the above equations.

6.28 Do yourself

6.29 $i = I_0 + I_{m1} \sin x - I_{m2} \cos 2x$



$I_{max} = I_0 + I_{m1} + I_{m2}$

$= (I_0 - I_{m2}) + I_{m1} + 2I_{m2}$

$I_b = I_0 - I_{m2}$

So, I_{max} (with a – c grid excitation) = $I_b + I_{m1} + 2I_{m2}$

I_{min} (with a – c grid excitation) = $I_b - I_{m1} + 2I_{m2}$

$I_{m1} = 0.5(I_{max} - I_{m1})$

$$I_{m2} = \frac{(I_{max} + I_{min}) - 2I_b}{4}$$

$$\therefore \quad i_{eff} = \sqrt{(2.5)^2 + \left(\frac{30}{\pi}\right)^2 + \left(\frac{30}{3\pi}\right)^2 + \left(\frac{30}{5\pi}\right)^2 + \left(\frac{30}{7\pi}\right)^2 + \left(\frac{30}{9\pi}\right)^2}$$

$$= \sqrt{1730.6459}$$

$$= 41.601 \text{ amp}$$

6.30 $I = I_b + I_{m1} \sin x + I_{m3} \sin 3x$

$I_b = 0.2$ amp

$I_{m1} = 0.07$ amp

$I_{n3} = 0.005$ amp

$= 0.28$

$i_{av9} \Rightarrow I_b = 0.2$

$i_{min} = I_b - I_{m1} + 2I_{m2}$

$= 0.2 - 0.07 + 2 \times 0.005$

$= 0.14$

$i_{max} = I_b + I_{m1} + 2I_{m2}$

$= 0.2 + 0.07 + 2 \times 0.005$

6.31

$$P = \frac{100 \times 20 + (-50) \times 10 + (25)(-5)}{2}$$

$= 687.5$ watt

6.32 Same as 31

6.33 $$pf = \frac{687.5}{\sqrt{\frac{100^2 + 50^2 + 25^2}{2}} \times \sqrt{\frac{20^2 + 10^2 + 5^2}{2}}}$$

$$= \frac{687.5}{81.0092 \times 16.20185}$$

$$= 0.5238$$

6.34 Same as 33.

6.35 (a) $P = \frac{100 \times 10 - 50 \times 5}{2} = 375$

(b) $$p.f = \frac{375}{\sqrt{\frac{100^2 + 50^2}{2}} \sqrt{\frac{10^2 + 5^2}{2}}}$$

$$= \frac{375}{79.05 \times 7.91}$$

$$= 0.5997$$

6.36

$V_{eff} = 81$ volt

So $V(t) = \sqrt{2} \times 81 \sin \omega t$

$p.f = 0.5238$

$\theta = 58.4125$

$\therefore i(t) = \sqrt{2} \times 16.20185 \sin (\omega t - 58.4125)$

6.38

$V = 100 \sin(\omega t + 30°) - 50 \sin(3\omega t + 60°) + 30 \cos 5\omega t$

$R = 6$ ohms

$C = 88.4\mu f$

$L = 0.01061$ H

$$z = 6 + j\left(\omega \times 0.01 - \frac{10^6}{\omega \times 88.40}\right)$$

$$= 6 + j\left(377 \times 0.01 - \frac{106}{377 \times 88.4}\right)$$

$$= 6 + j(-26.235) = 26.91 \angle -77.1178, \theta = 77.1178$$

$$v_{eff} = \sqrt{\frac{100^2 + 50^2 + 30^2}{2}}$$

$= 81.85$ volts

$$\therefore I_{rms} = \frac{81.85}{26.91} = 3.041 \text{ amp}$$

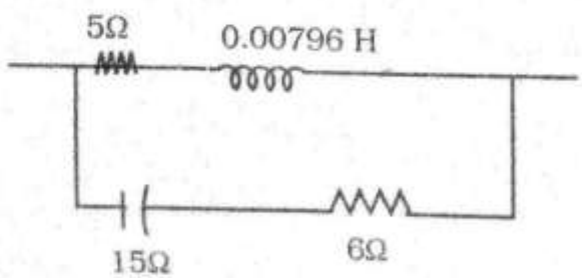
$\therefore P_{ower} = v_{vms} \times I_{rms} \cos \theta$

$= 81.85 \times 26.91 \times \cos(-77.187)$

$= 488.466$ watts

6.39 Similar to 38.

6.40



$X_L = 0.00796 \times 377$

$= 3\Omega$

$\therefore\ Z = (5 + i3\ \Omega)^{-1} + (6 - 15j)^{-1}$

$= 0.172 \angle -10.25\ \Omega$

$e = 100 \sin(\omega t + 30°) - 50 \cos(3\omega t - 30°)$

$e_{rms} = \sqrt{\dfrac{100^2 + 50^2}{2}}$

$= 111.8$ volt

$\therefore\ I_{rms} = \dfrac{111.8}{0.172} = 650$ amp

$I_{rms} = \sqrt{\dfrac{I_1^2 + I_2^2}{2}}$ and phase shift $= -10.25°$

So find the equation.

6.41 Ammeter reading = 650 amp

6.42 Power $= 111.8 \times 650 \times \cos 10.25$

$= 71510.2375$ watts

6.43 $pf = 0.9840$

$VA = 72670$ VA

6.44

$i_1 = 20 \sin(\omega t + 30) - 10 \sin(2\omega t - 30°) + 5 \sin(3\omega t - 40°)$

$i_2 = 15 \cos \omega t + 10 \cos(2\omega t - 60°) + 10 \cos(3\omega t + 50°)$

$i_{m1'} = 20(\cos 30° + j\sin 30°) = (8.66 + 5j) \times 2 = 17.32 + 10j$

$i_{m''1} = -10(\cos 30 - j \sin 30°) = -8.66 + 5j$

$im'''_1 = 5(\cos - 90° - j \sin 40°) = 3.83 - 3.2139\,j$

$\therefore\ i_{m2'} = 15$

$i_{m2''} = 10\,[\cos 60 - j \sin 60] = 5 - 8.66\,j$

$i_{m'2'} = 10\,[\cos 50 + j \sin 50] = 6.43 + 7.66\,j$

So, $i_{result} = i_{m1'} + i_{m2'}$

$i_{result} = i_{m1''} + i_{m''2}$

and $i_{result} = im'''_1 + i_{m'''2}$

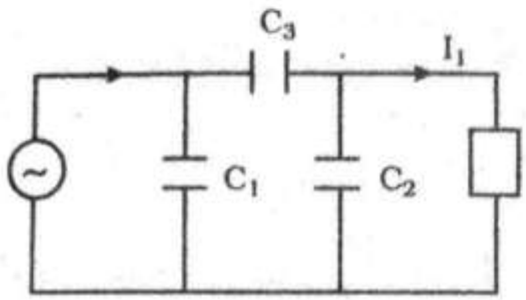
Find yourself.

45. Similar to 44.

Chapter–7

Coupled Circuits

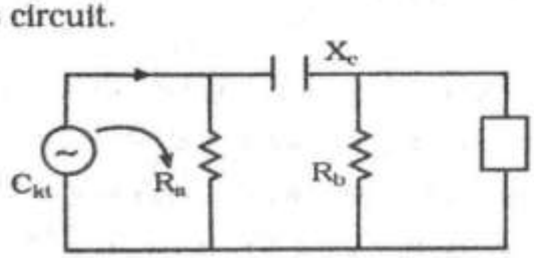
7.2. Let the voltage drop across c_1 is v_1



∴ Voltage developed across condenser c_1 per unit current is—

$$X_{coupling} = \frac{V_1}{I_2} = \frac{\frac{V_2}{X_1 + X_3} \cdot X_1}{V_2\left(\frac{1}{X_2} + \frac{1}{X_1 + X_3}\right)} = \frac{\frac{X_1}{X_1 + X_3}}{\frac{X_1 + X_3 + X_3}{X_2(X_1 + X_3)}} = \frac{X_1 X_2}{X_1 + X_2 + X_3}$$

7.3. When $Z_{coupling}$ in measured then CKt-2 is to be considered as circuit.



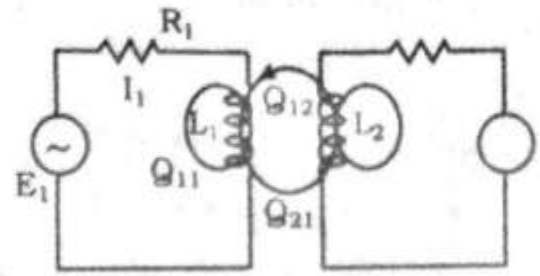
If V_b be the voltage developed across R_b then the coupling impedence between the two circuit is

$$Z_{coupling} = \frac{V_b}{I_1} = \frac{\frac{V_a}{R_b - j \times c} \times R_b}{V_a\left(\frac{1}{R_a} + \frac{1}{R_a - j \times c}\right)}$$

$$= \frac{\frac{R_b}{R_b} - j \times c}{\frac{R_b + R_a - j \times c}{R_a\,(R_b - j \times c)}} = \frac{R_a R_b}{R_a + R_b - j \times c}$$

$$= \frac{R_a^2 R_b + R_a R_b^2 + j\, R_a R_b \times X_c}{(R_a + R_b)^2 + X_c^2}$$

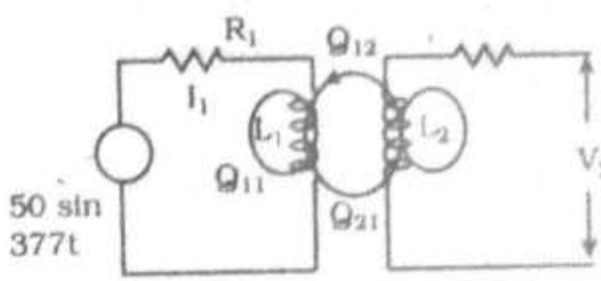
7.4.



(a) $M_{12} = N_2 \times \frac{\phi_{12}}{I} = \frac{500 \times 27500 \times 10^{-8}}{5} = 0.0275$ h

(b) $L_1 = N_1 \frac{\phi_1}{L_1} = \frac{50 \times 30{,}000 \times 10^{-8}}{5} = 3$ mh

7.5



$V = 50 \sin 3.77t \; \therefore \; \omega = 377$ rad/sec

$\therefore \quad X_m = \omega_m = 377 \times 0.02 = 7.54$ ohm

The voltage across the open circuited terminals of neighbouring coil is—

Now, $\frac{50}{\sqrt{2}} = I_1\left(10 + j377 \times \frac{1}{37.7} + 0\right)$

or, $I_1 = \frac{35.55}{10 + j10} = 2.5 \angle -50^\circ$

Now, $V_2 = I_2(j\omega L_2 + R_2) + I_1(j\omega m) = 0 + (2.5 \angle -45^\circ)(j377 \times 0.02)$

$= 2.5 \angle -45^\circ (7.54 \angle 90^\circ) = 18.85 \angle 45^\circ$

$\therefore$ The value of X_2 is 18.85 Volt

7.7. $V_1 = \frac{50}{\sqrt{2}} = 35.355 \angle 0°$ Volt

$I_1 = 2.5 \angle -45°$

$I_1R_1 = (2.5 \angle 45°)(10) = 25 \angle -45°$ Volts

$j \times L_1I_1 \quad = \left(j\,377 \times \frac{1}{37.7}\right)(2.5 \angle -45)$

$= (10 \angle 90°)(2.5 \angle -45°) = 25 \angle 45°$ Voltz

$j \times mI_1 = (j\,377 \times 0.02)(2.5 \angle -45°)$

$= (7.54 \angle 90°)(2.5 \angle -45) = 18.84 \angle 45°$

$E_{21} = -V_2 = 18.85 \angle 45° = -13.329 - j13.329 = 18.85 \angle -135°$

Now draw vector diagram by yourself.

7.8. $I_1 = 0.094$ h $\quad \therefore M = K\sqrt{L_1L_2}$

$L_2 = 0.0108$h $\quad = 0.0256$

$K_m = 0.805$

7.9. $N_1 = 1000$ turns, $\frac{\phi_1}{i_1} = 9400$ Max watts/amp $= 9400 \times 10^{-8}$ waber/amp

$N_2 = 338$ turns, $K = 0.805$

$L_1 = N_1\frac{\phi_1}{L_1} = 1000 \times 9400 \times 10^{-8} = 0.094$ h

$\frac{L_1}{L_2} = \frac{N_1^2}{N_2^2}\ L = \frac{N\phi}{i}$ again $Ni = R\phi$

$\therefore \quad L_2 = L_1 \times \frac{N_2^2}{N_1^2} = 0.094 \times \frac{338^2}{1000^2} = 0.0107$ h

$\therefore \quad M = K\sqrt{L_1L_2} = 0.805\sqrt{L_1L_2} = 0.0255$ h

1.10. Voltage of self-inductance in the L_2 coil considered as an induced voltage acts in a clack-wise direction around CKt–2 when $\frac{di_2}{dt}$ is considered positive. Since $M\frac{di_1}{dt}$ acts oppositely to $L_2\frac{di_2}{dt}$ in ckt – 2, M must be considered – ve if L_2 considered as + X_e. The general expression for voltage equilibrium in the ckt 2 is $-R_2i_2 + L_2\frac{di_2}{dt} - M\frac{di_1}{dt} = e_{ba}$

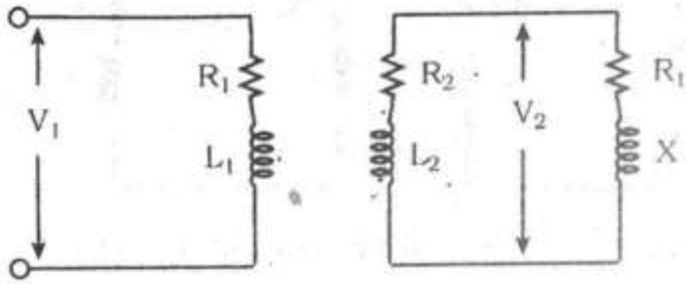
7.11. Here. $M = -3mh.\ Z_e = R_a + R_b + j(l_s + b_r - 2m)$

$$= (1 + 6) + j(1000)(0.004 + 0.009 - 0.006)$$

$$= 7 - j7 = 9.90 \angle -45^\circ$$

$$I = \frac{V}{Z_e} = \frac{40.5}{9.9} = 4.09 \text{ amp}$$

7.12.



The equaton are $+ I_1Z_1 + I_2Z_m = V$

$$I_1Z_m + I_2Z_2 = V$$

$$\therefore \quad I_1 = \frac{\begin{vmatrix} V & Z_m \\ V & Z_2 \end{vmatrix}}{\begin{vmatrix} Z_1 & Z_m \\ Z_m & z_2 \end{vmatrix}} = \frac{V(Z_2 - Z_m)}{Z_1Z_2 - Z_m^2}$$

$$I_2 = \frac{\begin{vmatrix} Z_1 & V \\ Z_m & V \end{vmatrix}}{\begin{vmatrix} Z_1 & Z_m \\ Z_m & Z_2 \end{vmatrix}} = \frac{V(Z_1 - Z_m)}{Z_1Z_2 - Zm^2}$$

$$I = I_1 + I_2 = \frac{V(Z_1 + Z_2 - 2Zm)}{Z_1Z_2 - Zm^2}$$

$$Z_e = \frac{V}{I} = \frac{Z_1Z_2 - Zm^2}{Z_1 + Z_2 - 2Zm^2}$$

$$= \frac{(35.5 \angle 84.7^\circ)(4.17 \angle 79.29^\circ) - (9.65 \angle 90^\circ)^2}{3.3 + j35.4 + 0.775 + j4.07 - j19.30}$$

$$= \frac{63.9 \angle 140.3}{20.578 \angle 78.58} = 3.1 \angle 61.7^\circ$$

$$I = \frac{V}{Zc} = \frac{50\angle 0}{3.1\angle 51.7^\circ} = 16.13 \angle -61.7^\circ \text{ A}$$

$$I_2 = V(Z_1 - Z_{12})(Z_1Z_2 - Zm^2)$$

$$= \frac{50\angle 0\,(3.3 + j35.4 - j9.65)}{63.9 \angle 140.3} = 4.41 \angle -27.2$$

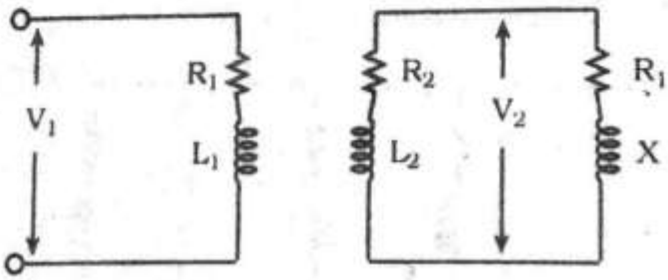
$$P = VI\cos\theta = 50 \angle 0 \times 16.13 \times \cos(61.7) = 382.352 \text{ watts}$$

7.13 $R_1 = 3.3$ omhs $\quad M = 0.0256$ H

$L_1 = 0.094$ H $\quad Z = 28.15$

$R_2 = 0.775$ ohms $\quad W = 377$ rad/sec

$L_2 = 0.0108$ H $\quad V_1 = 50 \angle 0$ volts



(a) $Z_{e1} = Z_1 - \frac{Zm^2}{Z_2 + Z} \cdot Z_2 + Z = (0.775 + j377)(0.0108) + 28.15$

$= 28.925 + j4.07 = 29.21 \angle 8^\circ$ omhs

$Z_{e1} = 3.3 + j35.4 + \frac{(9.65)^2 \angle 0}{29.21 \angle 80^\circ}$

$= 3.3 + j35.4 + 3.16 - j0.444$

$= 6.46 + j34.96 = 35.55 \angle 79.53$ omhs

$I_1 = \frac{50 \angle 0^\circ \angle 79.53^\circ}{35.55} = 1.407 \angle -79.53^\circ$ amp

$I_2 = \frac{-I_1 Zm}{Z_2 + Z} = \frac{(1.407 \angle -79.53 + 180)(9.65 \angle 90^\circ)}{29.21 \angle 8^\circ}$

$= 0.465 \angle 182.42$ amp

(b) $P_{in} = X_1 I_1 \cos\theta \left.\right]^{V_1}_{I_1}$

$= 50 \times 1.407 \times \cos 79.53 = 12.8$ watts

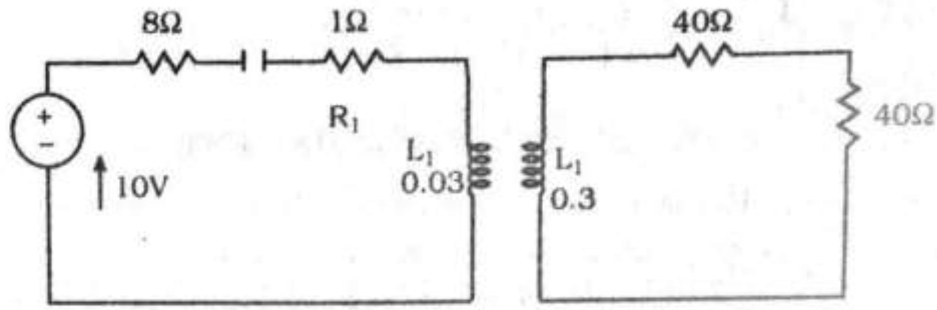
$P_{out} = V_2 I_2 \cos\theta \left.\right]^{V_2}_{I_2}, V_2 = 2I_2$

$= 28.15 \times 0.465 \angle 182.47 = 13.09 \angle 182.45^\circ$

$\therefore$ $P_{out} = 13.09 \times 0.465 \times \cos 182.47 = 6.08$ watts

$\therefore$ Efficiency of operation $= \frac{6.08}{12.8} = 0.475$ i.e., 47.5%

7.14 M = 0.275 henry, ω = 1667 rad/sec



(b) $2I_1 + 1I_1 - j8I_1 + j50.01I_1 + j458.425\ I_2 = 10$

or, $(3 + j42.01)\ I_1 + j458.425\ I_2 = 10$

or, $(100 + j500)\ I_2 + j1500I_2 + j458.425I_1 = 0$

or, $(100 + j5001)I_2 + j458.425I_1 = 0$

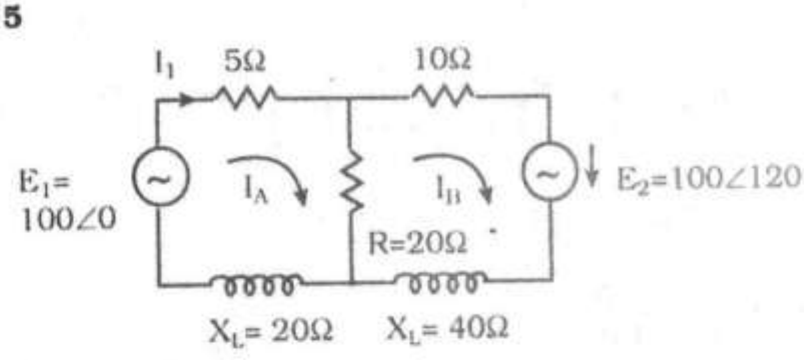
or, $j458.425I_1 + (100 + j5001)I_2 = 0$

$$I_1 = \frac{\begin{vmatrix} 10 & j458.425 \\ 0 & 100 + j5001 \end{vmatrix}}{\begin{vmatrix} 3 + j4201 & j458.425 \\ j458.425 & 100 + j5001 \end{vmatrix}} = 2.589\ \angle 0.09^\circ$$

$\therefore\quad I_1 = 2.589\ \angle 0.09^\circ$

$$I_2 = \frac{j458.425 \times 10}{19317.85\ \angle 88.94} = 0.237\ \angle 1.06$$

7.15



Now, $E_1 = Z_{11}\ I_A - Z_{12}I_B$

$E_2 = Z_{21}I_A + Z_{22}I_B$

$$\therefore\quad I_A = \frac{\begin{vmatrix} E_1 & -Z_{12} \\ E_2 & Z_{22} \end{vmatrix}}{\begin{vmatrix} Z_{11} & -Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix}} = \frac{E_1Z_{22} + Z_2Z_{12}}{Z_{11}Z_{22} - Z_{12}^2}$$

$$\text{and } I_B = \frac{\begin{vmatrix} Z_{11} & E_1 \\ Z_{21} & Z_2 \end{vmatrix}}{\begin{vmatrix} Z_{11} & -Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix}} = \frac{E_1 Z_{22} + E_2 Z_{12}}{Z_{11}Z_{22} - Z_{12}}$$

Where $Z_{11} = 5 + 20 + j20 = 25 + j20 = 32.018 \angle 38.66$

$Z_{22} = 10 + 20 + j40 = 30 + 40j = 50 \angle 50.13^\circ$ $Z_{12} = 20$ ohm

$$\therefore \quad I_A = \frac{(100 \angle 0)(50 \angle 53.1) + (100 \angle 120^\circ)(20)}{(32.016 \angle 38.66)(50 \angle 53.13) - 20^2}$$

$$= 3.653 \angle -34.94^\circ$$

$$I_B = \frac{(32.016 \angle 38.66)(100 \angle 120^\circ) + (100 \angle 0)(20)}{1662.077 \angle 105.70}$$

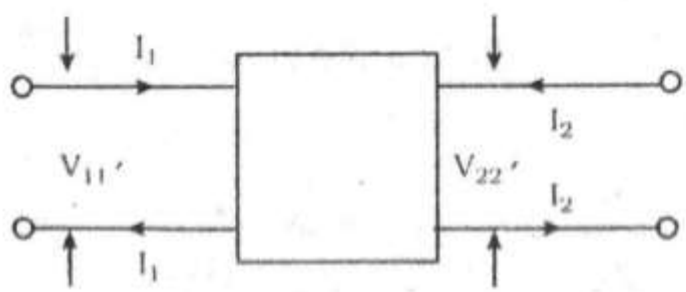
$$= \frac{-2982.09 + j1165.067 + 2000}{1662.077 \angle 105.7} = 0.917 \angle 24.43^\circ$$

Now, $I_1 = I_A = 3.653 \angle -34.94^\circ A$ $I_2 = I_B = 0.917 \angle 24.43^\circ A$

$$I_{12} = I_A - I_B = 2.995 - j2.092 - 0.335 - j0.379$$

$$= 3.28 \angle -48.84$$

7.16



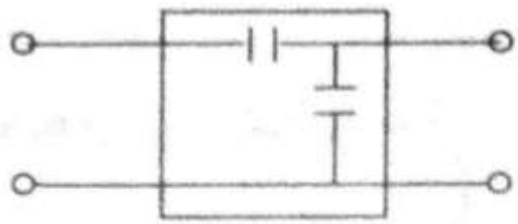
(a) $Z_{21} = \dfrac{V_{22}'}{I_1} = \dfrac{4\angle 10}{1\angle 90^\circ} = 4\angle -80^\circ$

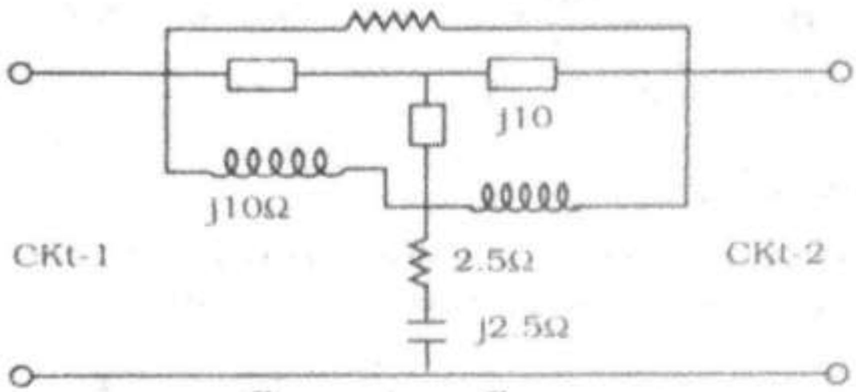
$$Z_{12} = \frac{V_{11}'}{I_2} = \frac{6\angle 0}{1.5\angle 90^\circ} = 4\angle -90^\circ$$

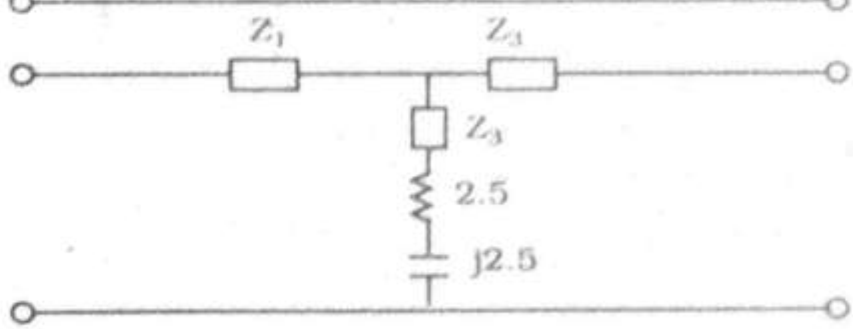
(b) $K = \dfrac{Z_{12}}{\sqrt{Z_{11}'Z_{2}'_{2}}}$ where $Z_1{}'_1 = \dfrac{E_1}{I_1} = \dfrac{6\angle 0}{1\angle 90^\circ} = 6\angle -90^\circ$

$$= \frac{4\angle -9j}{\sqrt{24\angle -90}} \quad Z_{22}' = \frac{6\angle 0}{1.5\angle +90} = 4\angle -90^\circ$$

$$= 0.816$$







$$Z_1 = \frac{j10 \times 20}{20 + j10 + j10} = \frac{200 \angle 90^\circ}{28.284 \angle 45^\circ}$$

$$= 7.071 \angle 45^\circ$$

$$Z_2 = \frac{j10 \times j10}{28.284 \angle 45^\circ} = \frac{100 \angle 180}{28.284 \angle 45^\circ}$$

$$= 3.536 \angle 135^\circ$$

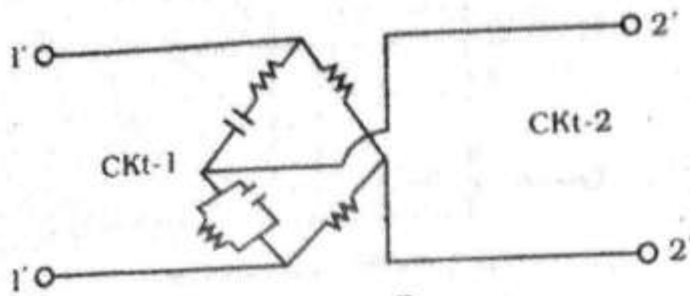
$$= -2.5 + 2.5$$

$$Z_3 = \frac{j10 \times 20}{28.284 \angle 45^\circ} = 7.071 \angle 45^\circ$$

Now, $Z_{12} = Z_2 + 2.5 - j2.5 = -2.5 + j2.5 + 2.5 - j2.5 = 0$

Now, $K = \frac{Z_{12}}{\sqrt{Z_{11}' Z_{22}'}} = 0$

7.17



Let us assume that K = 0 $\therefore$ $K = \frac{Z_{12}}{\sqrt{Z_1{}'_1 Z_2{}'_2}} = 0 \Rightarrow Z_{12} = 0$

$\Rightarrow \frac{V_{22}'}{Z_1} = 0 \Rightarrow V_2'_2 = 0$

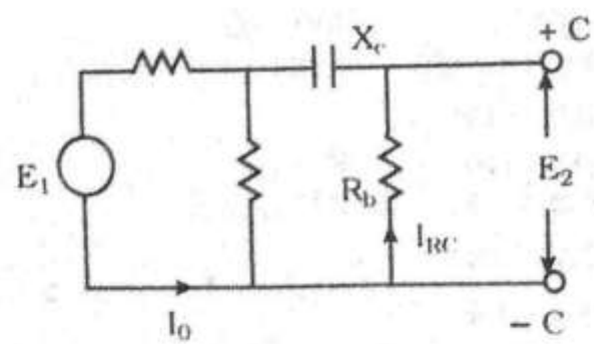
i.e.. there is no potential difference between 11°, the ckt is equivalent to a wheatstone bridge at balanced condition

$$\therefore \quad \frac{P}{Q} = \frac{R}{S} \Rightarrow PS = QR \Rightarrow \frac{j\frac{1}{\omega c_2}.R_2}{R_2 - j\frac{1}{\omega_1 c_2}} . R_a = R_b \left(R_1 - j\frac{1}{\omega c_1}\right) \; [R_a = R_b]$$

$$\text{or,} \quad R_1R_2 - \frac{1}{\omega^2 c_1 c_2} - j\frac{R2}{\omega c_1} - j\frac{\omega_2}{\omega c_2} = -j\frac{R_2}{\omega c_2}$$

$$\text{or,} \quad \frac{1}{\omega^2 c_1 c_2} = R_1R_2 \therefore \omega = \frac{1}{\sqrt{R_1R_2C_1C_2}} \text{ [Proved]}$$

7.18



If R_c drops from a to b V_2 drop from c to d. E_2 from c to d will be negative

$$\text{Now, } Z_0 = R_p + \frac{R_b\left(R_c - \frac{1}{\omega c_b}\right)}{R_b + R_c - \frac{1}{\omega c_b}} \left[I_0 = \frac{\mu E_g}{Z_0}\right]$$

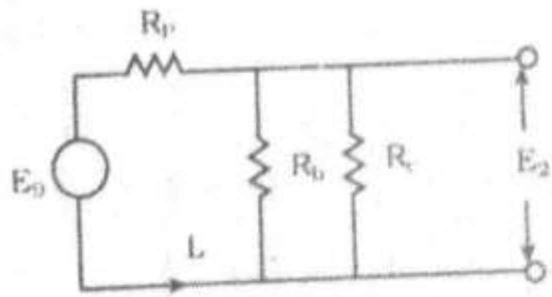
Now. $I_{RC} = \dfrac{R_b}{R_b + R_c - j\dfrac{1}{\omega C_b}}$

$\therefore$ from KVL we have $E_2 + I_{RC} \times R_c = 0$

or. $E_2 = -R_{RC} \times R_c$

or. $E_2 = -\left(\dfrac{R_b R_c}{R_b + R_c \dfrac{j}{\omega C_b}}\right) \times \mu E_g \times \dfrac{R_b + R_c - \dfrac{j}{\omega C_b}}{R_p\left(R_b + R_c - \dfrac{j}{\omega C_b}\right) + R_b\left(R_c - \dfrac{j}{\omega C_b}\right)}$

$$= -\frac{\mu E_g R_b R_c}{R_p R_b + R_p R_c + R_b R_c - j\dfrac{R_p + R_b}{\omega C_b}}$$



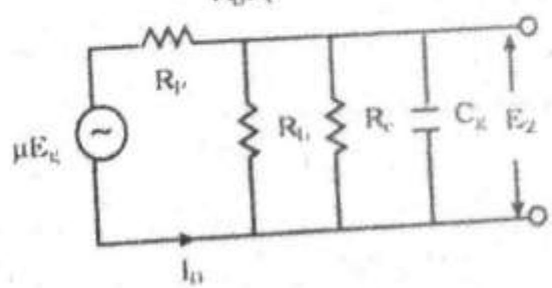
$Z_0 = R_b + \dfrac{R_b R_c}{R_b + R_c} = \dfrac{R_p(R_b + R_c) + R_b R_c}{R_b + R_c}$ $\therefore I_0 = \dfrac{\mu E_g}{Z_0}$

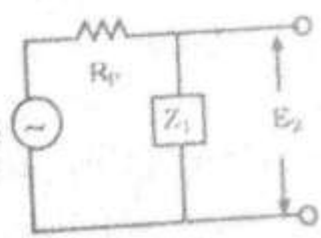
$\therefore$ $I_{RC} = \dfrac{R_b}{R_b + R_c} I_0$. Now from KVL we have $I_{RC} + E_2 = 0$

or. $E_2 = -R_c \times \dfrac{R_b}{R_b + R_c} \cdot \mu E_g \times \dfrac{R_b + R_c}{R_p(R_b + R_c) + R_b R_c}$

$$= \frac{-\mu E_g R_b R_c}{R_p(R_b + R_c) + R_b R_c}$$

$$= \frac{-\mu E_g}{R_p \dfrac{R_b + R_c}{R_b R_c} + 1} \text{ (Proved)}$$





$$\frac{1}{Z} = \frac{1}{R_b} + \frac{1}{R_c} + \frac{1}{-\frac{1}{j\omega c_g}} \therefore Z_0 = R_p + \frac{R_b R_c \left(-j\frac{1}{\omega c_g}\right)}{R_b R_c + R_c \left(-j\frac{1}{\omega c_g}\right)} + R_b \left(-j\frac{1}{\omega c_g}\right)$$

$$= R_p + Z \therefore I_0 = \frac{\mu E_g}{Z_0} = \frac{\mu E_g}{R_p} \text{ from KVL we have}$$

$$E_2 = -ZI_0 \frac{-R_b Rc \left(-j\frac{1}{\omega c_g}\right)}{R_b R_c + R_b \left(-j\frac{1}{\omega c_g}\right) + R_c \left(-j\frac{1}{\omega c_g}\right)}$$

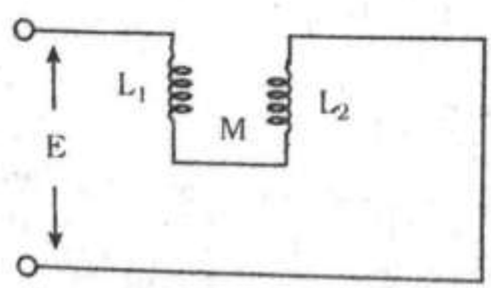
$$\times \frac{E_9}{\dfrac{R_p + R_b R_c \left(-j\frac{1}{\omega c_g}\right)}{R_b R_c + R_b\left(-j\frac{1}{\omega c_g}\right) + R_c}}$$

$$= \frac{-\mu E_9 E_b R_c \left(-j\frac{1}{\omega c_g}\right)}{R_p \{R_b R_c + R_b \left(-j\frac{1}{\omega c_g}\right) + R_c \left(-j\frac{1}{\omega c_g}\right) + R_b R_c \left(-j\frac{1}{\omega c_g}\right)}$$

$$= \frac{-\mu E_g R_b R_c \left(-j\frac{1}{\omega c_g}\right)}{R_p R_b R_c + R_p \left(-j\frac{R_b + R_c}{\omega c_g}\right) + R_b R_c \left(-j\frac{1}{\omega c_g}\right)}$$

$$= \frac{-\mu E_9}{\frac{R_p}{-j\frac{1}{\omega c_g}} + R_p \frac{R_b + R_c}{R_b - R_c} + 1} = -\frac{\mu E_g}{jR_p \omega c_g + R_p \frac{R_b \times R_c}{R_b + R_c} + 1}$$

7.23



$L_1 = 60$ mh $= 0.06$ h

$L_2 = 30$ mh $= 0.03$ h

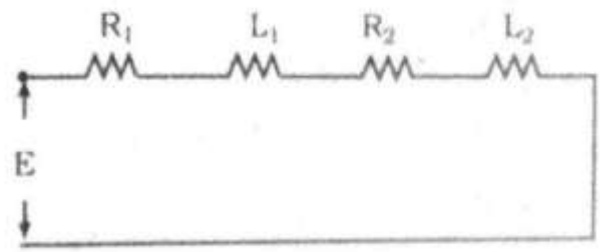
Now. L_T (add) = $L_1 + L_2 + 2m$ or. $0.12 = 0.06 + 0.03 + 2m$

$\therefore \quad 2m = 0.03$ h

(a) L_T (sub) = $L_1 + L_2 - 2m = 0.06 + 0.03 - 0.03 = 60$ mh

(b) $K = \dfrac{M}{\sqrt{L_1 L_2}} = \dfrac{0.03/2}{\sqrt{0.06 \times 0.03}} = 0.354$

7.24



For additive combination = $I^2(R_1 + R_2) = 200$

$\therefore \quad R_1 + R_2 = \dfrac{200}{5^2} = 80$ ohm

Now, magnitude of Z_0 is = $\dfrac{100}{5} = 20$

$\therefore \quad 20 = \sqrt{(R_1 + R_2)^2 + (L_1 + L_2 + 2m)^2 \times (2\pi 69.5)^2}$

$[\, Z = \sqrt{R^2 + X_L^2}\,]$

or. $400 = 64 + (L_1 + L_2 + 2m)^2 (2\pi \times 69.5)^2$

$\therefore \quad L_1 + L_2 + 2m = 0.04198$

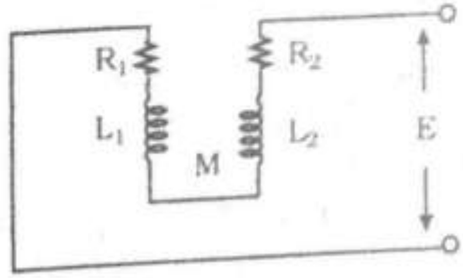
For subtrative combination $Z = \dfrac{100}{8} \sqrt{8^2 + (2\pi \times 69.5\,(L_1 + L_2 - 2m))}$

$\therefore \quad L_1 + L_2 - 2m = 0.022$

$4m = 0.04198 - 0.022$

$\therefore \quad M = 0.005$ h = 5 mh

7.25



$R_1 = R_2 = 4$ omh

$$V = IR_1 + j\omega L_1 I + j\omega MI$$

$$\frac{V}{I} = R_1 + j(L_1 + M)\omega$$

or, $\frac{26.05}{5} = 4 + j\omega(L_1 + 0.005)$

or, $(7.21)^2 = 4^2 + \omega^2(L_1 + 0.005)^2$

or, $(7.21)^2 = 4^2 + (2\pi \times 69.5)^2 (L_1 + 0.005)^2$

$\therefore \quad L_1 = 8.737$ mh

Now, $L_1 + L_2 + 2M = 0.04198$

$\therefore \quad L_2 = 0.04198 - 2 \times 0.005 - 0.008737$

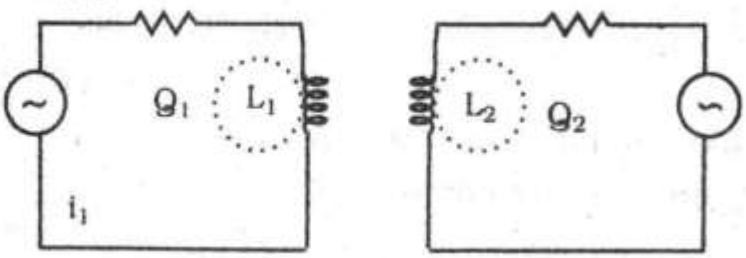
$= 23.249$ mh

Now,

$$Z_2 = \sqrt{4^2 + (L_2 + M)^2 (w^2)} = 12.96 \text{ ohm}$$

$\therefore \quad V_2 = IZ_2 = 5 \times 12.96 = 64.8$ Volts

(b) $K = \frac{M}{\sqrt{L_1 L_2}} = 0.35$ Ans.



$L_1 = 0.1$ h, $L_2 = 0.05$ h $M = K\sqrt{L_1 L_2}$

$= 0.56\sqrt{0.1 \times 0.05}$

$= 0.0356$ h

Now the voltage induced in the 0.05 h winding as a result of L_1 is $- M\frac{di}{dt} = 0.0396 - \frac{d}{dt}(10 \sin 377t)$

$= 0.0396 \times 10 \times 377 \cos 377t$

$$X_2 = Z_m I_1 = wMI_1 = 377 \times 0.4 = X\frac{10}{\sqrt{2}} = 105.56 \text{ V}$$

$\therefore \quad$ effective value $= -\frac{142.292}{\sqrt{2}} = 105.56$ V

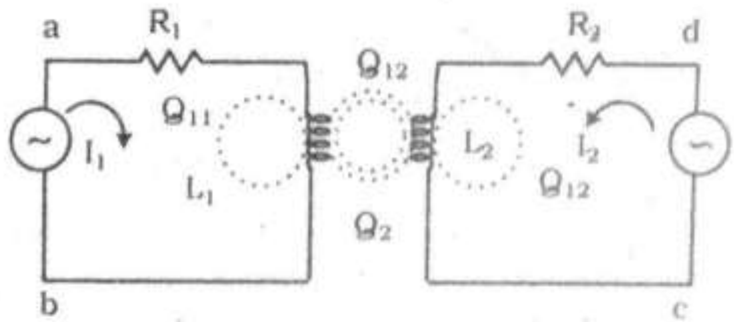
Now, the induced voltage in 0.1 henry winding is—

$$L_1 = \frac{di_1}{dt} = 0.1 \times 10 \times 377 \cos 377t = 377 \cos 377t$$

$$\therefore \quad \text{r.m.s value} = \frac{377}{\sqrt{2}} = 266.58 \text{ V}$$

$$\text{Again } V_1 = I_1 \times L_1 = \frac{10}{\sqrt{2}} \times 377 \times 0.1 = 266.58 \text{ V}$$

7.27



$$M = K_M\sqrt{L_1L_2} = 0.4\sqrt{0.056 \times 0.07} = 0.025 \text{ h. } \dot{M} \text{ will be neg}$$

$$Z_1I_1 - Z_MI_2 = E_{ba}$$

$$\text{and } -Z_MI_1 + Z_2I_2 = E_{cd}$$

$$I_1 \quad = I_{ba} = \frac{\begin{vmatrix} E_{ba} & -Z_M \\ E_{cd} & Z_2 \end{vmatrix}}{\begin{vmatrix} Z_1 & -Z_M \\ -Z_M & Z_2 \end{vmatrix}} = \frac{E_{ba}Z_2 + E_{cd}Z_M}{Z_1Z_2 - Z_M^2}$$

$$= \frac{100\angle 0 \times (72 + j0.07 \times 1131) + (50\angle -90^\circ)(j113 \times 0.025)}{(50 + j0.056 \times 1131)(72 + j0.07 \times 1131) - (j1131 \times 0.025)^2}$$

$$= \frac{7199.846 + j7917.528 + 1413.75}{(80.694\angle 51.71)(107.013\angle 47.72) + 28.275^2}$$

$$= \frac{11699.26\angle 42.59}{1414.83 + j8518.614 + 28.275^2} = \frac{116999.26\angle 42.56}{3540.811\angle 94.312}$$

$$= 1.37\angle -51.54$$

$$I_2 \quad = I_{cd} = \frac{\begin{vmatrix} Z_1 & E_{ba} \\ -Z_M & E_{cd} \end{vmatrix}}{\begin{vmatrix} Z_1 & -Z_M \\ -Z_M & Z_2 \end{vmatrix}} = \frac{Z_1E_{cd} + Z_MZ_{ba}}{Z_1Z_2 - Z_M^2}$$

$$= \frac{(80.694 \angle 51.71)(50 \angle -90^\circ) + (28.275 \angle 90^\circ)(100 \angle 0)}{8540.811 \angle 94.312^\circ}$$

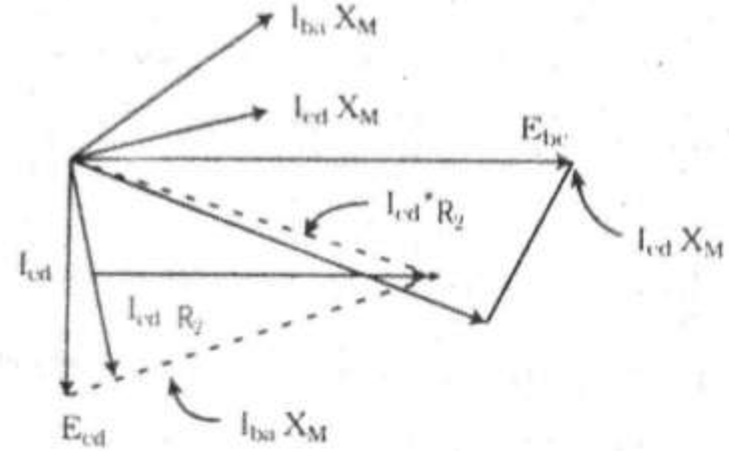
$$= \frac{3166.774 - j2500 + j2827.5}{8540.81 \angle 94.312} = \frac{3183.656 \angle 5.9}{8540.81 \angle 94.312}$$

$$= 0.373 \angle -88.4^\circ$$

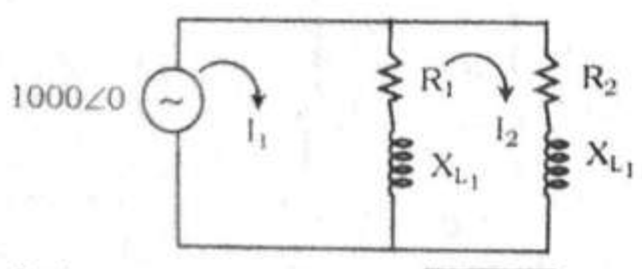
(b) $P_{ba} = V_{ba} I_1 \cos\theta = 100 \times 1.37 \times \cos 51.54 = 85.21$ watt

$P_{cd} = V_{cd} I_2 \cos\theta = 50 \times 0.373 \cos(-90 - (-88.4))$

$= 17.64$ watts



(c)



$R_1 = 2$ ohm $\qquad Z_M = K\sqrt{X_{L1} \times L_2}$

$X_{L1} = 3$ ohm $\qquad = 0.8 \times \sqrt{3 \times 12}$

$X_{L2} = 12$ ohm $\qquad = 4.8$ ohm

Since mmf's are additive ∴ M will be positive.

$Z_1 = 2 + j3 = 3.606 \angle 56.31$

$Z_2 = 5 + j12 = 13 \angle 67.38^\circ$

$Z_M = j4.8 = 4.8 \angle 90^\circ$

Now, $I_1 Z_1 + Z_2 Z_M = X$

and $I_1 Z_M + I_2 Z_2 = X$

$$\therefore \quad I_1 = \frac{\begin{vmatrix} V_1 & Z_M \\ V_1 & Z_2 \end{vmatrix}}{\begin{vmatrix} Z_1 & Z_M \\ Z_M & Z_2 \end{vmatrix}} = \frac{V(Z_2 - Z_M)}{Z_1 Z_2 - Z_M^2}$$

$$= \frac{(100 \angle 0)(5 + j12 - 4.3j)}{(3.606 \angle 56.31)(13 \angle 67.38) - (4.8 \angle 90^\circ)^2}$$

$$= \frac{876.584 \angle 56.2^2}{-22 + j39 + 23.04} = \frac{876.584 \angle 55.22}{39.112 \angle 94.34}$$

$$= 22.41 \angle -39.12$$

$$I_2 = \frac{\begin{vmatrix} Z_1 & V \\ Z_M & V \end{vmatrix}}{\begin{vmatrix} Z_1 & Z_M \\ Z_M & Z_2 \end{vmatrix}} = \frac{V(Z_1 - Z_M)}{Z_1 Z_2 - Z_M^2} = \frac{(100 \angle 0^\circ)(2 + j3 - j4.8)}{39.112 \angle 94.34}$$

$$= \frac{269.07 \angle -42}{39.112 \angle 94.34} = 6.879 \angle -136.34^\circ A$$

Power supplied conductively branche (2) is

$= VI_2 \cos\theta = 100 \times 6.879 \times \cos(-136.34)$

$= -497.66$ watts

Power dissipated in branch (2) is $= I_2^2 R_2$

$= (6.879)^2 \times 5 = 236.6$ watts

$\therefore$ Power supplied electromagneticcelly is

$= P_{conductivity} + P_{dissipated}$

$= 497.66 + 236.6 = 734.26$ watts

Again, power supplied electromagnetically in branch (1) is—

$P_{elect} = -P_{conductivity} + P_{dissipated} = -VI\cos\theta + I_1^2 R$

$= -1738.63 + 1304.42 = -734.21$ watts

The voltage drop accross only the inductance of branch (2) is

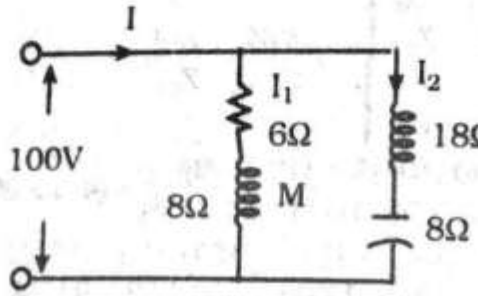
$V_2 = X_{L2} I_2 + I_1 Z_M \cdot V_2$

$= (12 \angle 90^\circ)(6.879 \angle 136.34) + (22.41 \angle -39.12) \times (4.8 \angle 90^\circ)$

$= 57 - j59.72 + 67.87 + j83.454 = 127.1 \angle 10.76^\circ$

$\therefore \quad I_2$ lags V_2 by $(10.76 - (-136.34)) = 147.1^\circ$

7.29



$X_M = K\sqrt{X_{L1}X_{L2}} = 0.5\sqrt{8 \times 18} = 6$ ohm

Here, M will be negative

Now, $Z_1I_1 - Z_MI_2 = V$ Where $Z_1 = 6 + j8$

$- Z_MI_1 + Z_2I_2 = V$ $Z_2 = -j8 + j18 = j16$

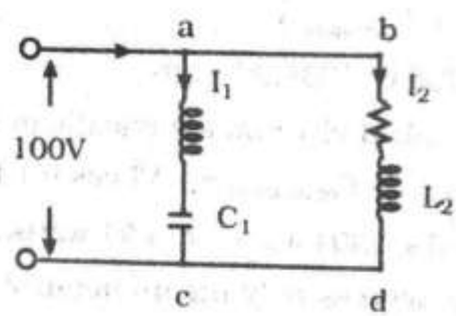
$$\therefore \quad I_1 = \frac{\begin{vmatrix} V & -X_M \\ V & Z_2 \end{vmatrix}}{\begin{vmatrix} Z_1 & -Z_M \\ -Z_M & Z_2 \end{vmatrix}} = \frac{V(Z_2 + Z_M)}{Z_1Z_2 - Z_M^2}$$

$$= \frac{(100 \angle 0)\{(-j8 + j18) + j6\}}{(6 + j8)(-j8 + j18) - (j6)^2}$$

$$= \frac{1600 \angle 90^\circ}{j60 - 80 + 36} = \frac{1600 \angle 70^\circ}{74.404 \angle 126.25}$$

$$= 21.5 \angle -36.25^\circ A$$

7.30



Now,

$X_{L1} = 5$ ohm, $X_{c1} = 5$ ohm

$X_{L2} = 5$ ohm, $X_M = 4$ ohm

R = 10, M is – ve

$\therefore \quad (5j - 5j)\ I_1 - 4jI_2 = 100 \angle 0^\circ$

or. $(4 \angle -90^\circ) I_2 = 100 \angle 0$

$\therefore \quad I_2 = 25 \angle 90^\circ$

Again.

$$(10 + j5)I_2 - j4I_1 = 100 \angle 0$$

or. $(10 + j5)(j25) - 4jI_1 = 100$

or. $250j - 125 - 4jI_1 = 100$

or. $4jI_1 = 250j - 125 - 100$

$$I_1 = 84.085 \angle 41.9872$$

From KVL we have now.

$$V_{bc} + V_{ca} + V_{ab} = 0$$

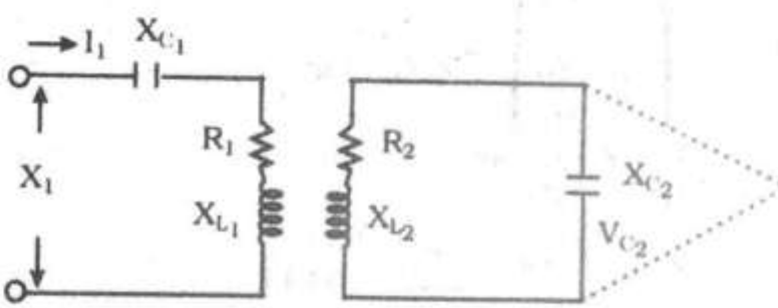
or. $V_{bc} = V_{ac} - V_{ab} = R_2I_2 - (X_{L1}I_1 - X_MI_2)$

$= 10 \times (25 \angle 90^\circ) - \{j5(84.085 \angle 42^\circ - j4(25 \angle 90^\circ)\}$

$= 250 \angle 90^\circ - \{-281.319 + j312.437 + 100\}$

$= 191.8 \angle -19^\circ$ Vots

7.31



R_1	= 4 ohms	R_2	= 100 ohms
X_{L1}	= 40 ohms	X_{L2}	= 100 ohms
X_{c1}	= 40 ohms	X_{c2}	= 120 ohms
X_M	= 500 ohms	V_1	= 100 volts

Since the direction of the mutual flux in the secondary of the circuit ϕ_{12} and current I_2 same. $\therefore$ M is positive.

Now,

$$Z_1 = 4 + j40 - j40 = 4 \text{ ohm}$$

$Z_2 = 10 + j100 - 120j = 10 - j20 = 22.36 \angle 63.43^\circ$;

$Z_M = j50 = 50 \angle 90^\circ$

Now.

$Z_1I_1 + Z_MI_2 = V_1$ Here $Z_2 = Z_2 + Z$ of book

$Z_MI_1 + Z_2I_2 = 0$

$$I_2 = \frac{\begin{vmatrix} Z_1 & V_1 \\ Z_M & 0 \end{vmatrix}}{\begin{vmatrix} Z_1 & Z_M \\ Z_M & Z_2 \end{vmatrix}}$$

$$= \frac{-V_1Z_M}{Z_1Z_2 - Z_M^2} = \frac{-(100\angle 0^\circ)(50\angle 90^\circ)}{(4)(22.36\angle -63.43) - (50\angle 90^\circ)^2}$$

$$= \frac{5000\angle 90^\circ}{40 - j80 + 2500} = \frac{-5000\angle 90}{2541.26\angle -1.8} = -1.9675\angle 91.8$$

Now, $V_{C2} = I_2X_{C2} = (1.9675\angle 91.8)(120\angle -90^\circ) = 236.1\angle 1.8^\circ$

7.32

From the previous problem we have—

$$I_1 = \frac{\begin{vmatrix} V_1 & Z_M \\ 0 & Z_2 \end{vmatrix}}{\begin{vmatrix} Z_1 & Z_M \\ Z_M & Z_2 \end{vmatrix}} = \frac{V_1Z_2}{Z_1Z_2 - Z_M^2}$$

$$\therefore \quad Z_{e1} = \frac{V_1}{I_1} = \frac{Z_1Z_2 - Z_M^2}{Z_2}$$

$$= Z_1 - \frac{Z_M^2}{Z_2} = 4 - \frac{(50\angle 90^\circ)}{22.36\angle -63.43}$$

$$= 4 - 111.8\angle 243.43$$

$$= 4 + 50 + j100 = 54 + j100 = 113.65\angle 61.63^\circ$$

The reflected impedence into the primary $= \dfrac{Z_M^2}{Z_2}$

$= 111.8\angle 243.43$

$= 50 + j100 = 111.8\angle 63.43$

Now, the value of reflected reactance into the primary = j100 which is inductive.

(33) and (34) For reference, hence.

7.33

$R_1 = 3.3\Omega$ $\qquad R_2 = 0.094$ H

$L_1 = 0.094$ H $\qquad X_L = \omega L = 35.4$

$C_1 = 83 \times 10^{-6} f$ $\qquad X_c = \frac{1}{\omega c} = 31.95$

So. $Z_1 = 3.3 + 35.1j - 31.95j$

$= 33 + 3.44j$

$Z_2 = 0.775 + j4.07$

$Z_M = jM \mid Z = 14.5 + j21.2$

$$Z_{e1} = Z_1 - \frac{Z_M^2}{Z_2 + Z} = 3.3 + 3.44j + \frac{M^2}{15.28 + j25.3}$$

$= 3.3 + 3.44j + M^2(0.0175 - 0.03j)$

$= 3.3 + 3.44j + M^2 \times 0.0175 - M^2 \times 0.03\,j$

$= (3.3 + M^2 \times 0.0175) + j(3.44 - M^2 \times 0.03)$

$= \sqrt{(3.3 + M^2 \times 0.0175)^2 + (3.44 - M^2 \times 0.03)^2}$

$$\tan^{-1} \frac{3.44 - M^2 \times 0.03}{3.3 + M^2 \times 0.0173}$$

So. $\frac{3.44 - M^2 \times 0.03}{3.3 + M^2 \times 0.0175} = 0$

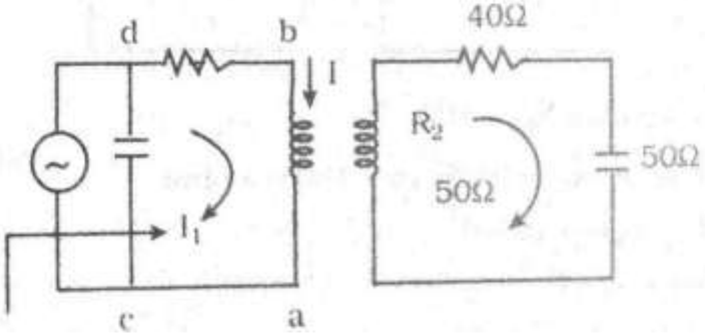
$\frac{3.44}{0.03} = M^2$

So $M = 10.708$ | Here $M = \omega m'$

So. $M' = \frac{10.708}{377} = 0.0284$ H

7.34 Do yourself

7.35



Let the capacitive reactance of c_{11} be X.

$$\text{Now, } Z_{cabd} = Z_1 - \frac{Z_M^2}{Z_2} = 10 + j10 - \frac{(j20)^2}{10 + j50 - j50}$$

$$= 10 + j10 + 10$$

$$= 20 + j10 = 22.36 \angle 20.57^\circ$$

$$\text{Now, } Z_{01} = \frac{(-jX)Z_{cabd}}{-jX + Z_{cabd}} = \frac{(X \angle -90^\circ)(22.36 \angle 26.57^\circ)}{-jX + 20 + j10}$$

$$= \frac{22.36\, x \angle -63.43}{\sqrt{20^2 + (10 - x)^2} \tan^{-1} \frac{10 - x}{20}}$$

$$= \frac{22.36x}{\sqrt{20^2 + (10 - x)^2}} \times \angle -63.43 - \tan^{-1} \frac{10 - x}{20}$$

For pure resistance phase angle will zero

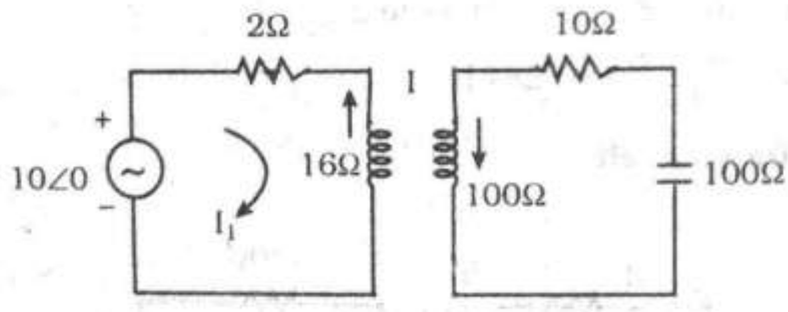
$$\therefore \quad \tan^{-1} = \frac{10 - x}{20} = -63.43 \text{ or, } \frac{10 - x}{20} = \tan(-63.43)$$

$$\therefore \quad x = 50 \text{ ohm}$$

$\therefore$ The value of the pure resistance

$$= \frac{22.36 \times 50}{\sqrt{20^2 + (10 - 50)^2}} = 25 \text{ ohm}$$

7.36



$$X_{L1} = 16 \text{ ohm } X_{L2} = 100$$

$$X_M = K\sqrt{X_{L1} X_{L2}} = 0.05\sqrt{16 \times 100} = 2 \text{ ohm}$$

$$\text{Now, } Z_1 I_1 + Z_M I_2 = 10 \angle 0^\circ$$

$$Z_M I_1 + Z_2 I_2 = 0$$

$$\therefore \quad I_2 = \frac{\begin{vmatrix} Z_1 & 10\angle 0 \\ Z_M & 0 \end{vmatrix}}{\begin{vmatrix} Z_1 & Z_M \\ Z_M & Z_2 \end{vmatrix}}$$

$$= \frac{-Z_M(10\angle 0)}{Z_1Z_2 - Z_M^2} = \frac{-j2(10)}{(2+j16)(10+j100-j100-(2j)^2)}$$

$$= 0.1236\angle -171.47 \text{ A}$$

$$\therefore \quad V_{c2} = X_{c2} \times I_2 = 100 \times 0.1236 = 12.36 \text{ volt}$$

(b) At resonance, the impedence of CKt – 1 will be.

$$Z_1 = 2 + j16 - j16 = 2 \text{ ohm}$$

$$\therefore \quad I_1 = \frac{-j(2\times 10)}{2\times 10-(2j)^2} = \frac{20\angle -90^\circ}{24} = 0.833\angle -90^\circ$$

$$\therefore \quad V_{c22} = 100 \times 0.833 = 83.3 \text{ Volt}$$

(c) At resonance the equation are

$$ZI_1 + Z_MI_2 = 10$$

and $Z_MI_1 + 10I_2 = 0$

$$I_2 = \frac{\begin{vmatrix} Z & 10 \\ Z_M & 0 \end{vmatrix}}{\begin{vmatrix} Z & Z_M \\ Z_M & 0 \end{vmatrix}}$$

$$= \frac{-10\,Z_M}{20 - Z_M} = \frac{-10\times Z_M\angle 90^\circ}{20-(X_M\angle 90^\circ)} = \frac{-10X_M\angle 90^\circ}{20+X_M}$$

$$\therefore \quad V_{c22} = (100\angle -90^\circ)\left(\frac{-10X_M\angle 90^\circ}{20+X_M^2}\right) = \frac{-1000X_M}{20+X_M^2}$$

$$= \frac{-1000}{\frac{20}{X_M}+X_M} = \frac{-1000}{0}$$

When denomenator $D = \frac{20}{X_M} + X_M$

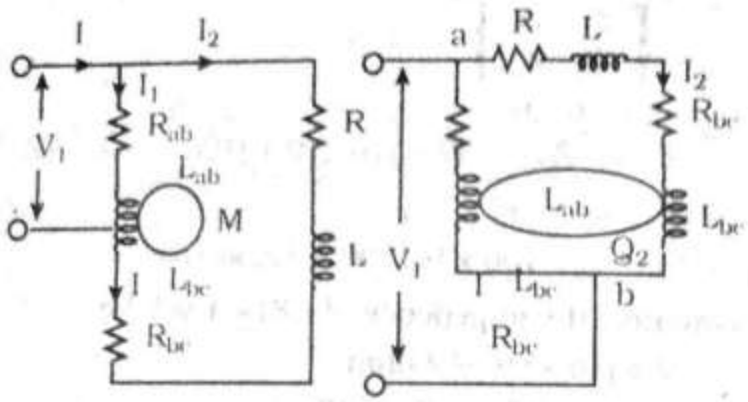
V_{c22} will be maximum it's D will be minimum

$$\therefore \quad \frac{dD}{dX_M} = -\frac{20}{X_M^2} + 1 = 0$$

or. $X_M^2 = 20$ or. $X_M = \sqrt{20} = 4.472$ ohm

$$\therefore \quad X_{c22} = \frac{1000\times 4.472}{20+20} = 111.8 \text{ Volt}$$

37.



Here M will be negative

∴ The general differential equations are—

$$R_{ab} I_1 + L_{ab}\frac{di_1}{dt} - M\frac{di_2}{dt}$$

$$= V_1 \; (R + R_{bc})\, i_2 + (L + L_{4c})\,\frac{di_2}{dt} - M\frac{di_1}{dt} = V$$

7.38 Let

$$Z_{ab} = R_{ab} + jwL_{ab}$$

$$Z_{bc} = R_{bc} + jwL_{4c}$$

$$Z = R + jwL$$

$$Z_M = jw_M$$

Now the equations become—

$$Z_{ab} I_1 - Z_M I2 = V_1$$

$$-Z_M I_1 + (Z_{4c} + Z)\, I_2 = V_1$$

$$\therefore \quad I_1 = \frac{\begin{vmatrix} V_1 & -Z_M \\ V_1 & Z_{bc} + Z \end{vmatrix}}{\begin{vmatrix} Z_{ab} & -Z_M \\ -Z_M & Z_{bc} + Z \end{vmatrix}} = \frac{V_1\,(Z_{bc} + Z + Z_M)}{Z_{ab}\,(Z_{4c} + Z) - Z_M^2}$$

$$\therefore \quad I_2 = \frac{\begin{vmatrix} Z_{ab} & V_1 \\ Z_M & V_1 \end{vmatrix}}{\begin{vmatrix} Z_{ab} & -Z_M \\ -Z_M & Z_{bc} + Z \end{vmatrix}} = \frac{V_1\,(Z_{ab} + Z_M)}{Z_{ab}\,(Z_{4c} + Z) - Z_M^2}$$

7.39

$Z_{ab} = 4 + j377 \times 0.07 = 4 + j26.39$

$= 26.69 \angle 81.38$ ohm

$Z_{bc} = 0.5 + j377 \times 0.01 = 0.5 + j3.77 = 3.8 \angle 82.45^\circ$ ohm

$Z = 10 + j0 = 10$ ohm

$Z_M = j377 \times 0.02 = 7.54 \angle 90^\circ$ ohm

$$I_1 = \frac{(100 \angle 0)(0.5 + j3.77 + 10 + j7.5)}{(26.8 \angle 81.38)(0.5 + j3.77 + 10) - (j7.54)^2}$$

$$I_2 = \frac{(100 \angle 0)(4 + j26.39 + j7.54)}{292.16 \angle 90.12}$$

$$= \frac{2316.5 \angle 83.28}{292.16 \angle 90.16} = 1169 \angle -6.84 \text{ ohm}$$

$I = I_1 + I_2 = 5.28 \angle -43 + 1169 \angle -6.89$

$= 16.25 \angle -17.9^\circ$ ohm

$\therefore$ Total power supplied $= V_1 I \cos 417.9) = 1545.9$ watts

Power dissipated in circuit $1 = I_1^2 R_{ab} = 111.5$ watts

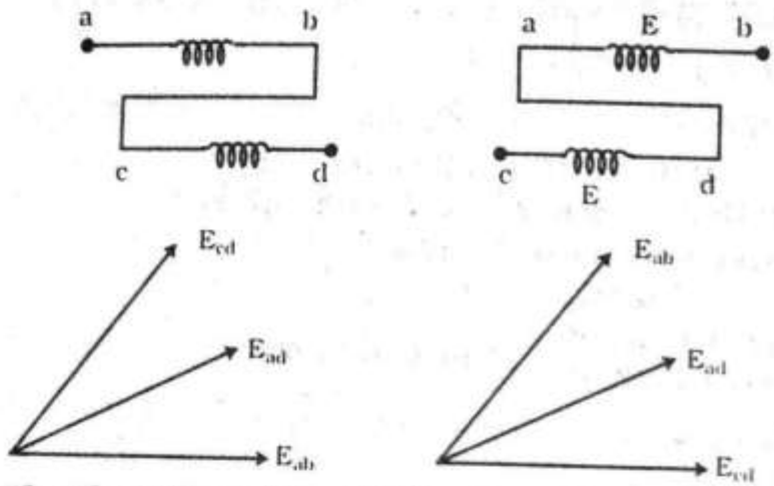
" " " $2 = I_2^2 (R_{bc} + R) = 11.69^2 \times$

$(0.5 + 10) = 1443.9$ watts

40, 41, 42, 43, 44, 45 are out of syllabus, it will be studied in higher level and term.

Chapter–8
Balanced Polyphase Circuits

8.1



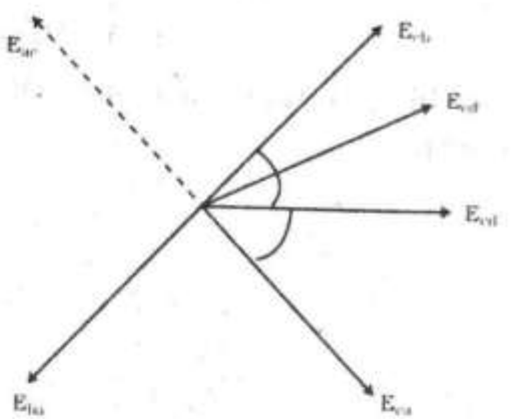
$E_{ac} = E^2 + 2^2 + 2E^2 \cos 60^\circ = 2E^2 (1 + \cos 60^\circ) = 2E^2.2 \cos^2 30^\circ$

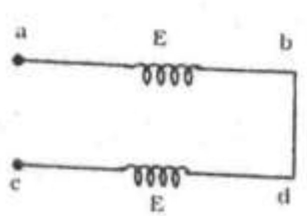
$E_{cb} = 2E \cos 30^\circ = 1.732\ E$

$E_{ab} = 2E \cos 30^\circ = 1.732\ F$

$\therefore \quad E_{ad} = F_{cd}$

8.2





(a) $\bar{E}_{ca} = \bar{E}_{cd} + \bar{E}_{bc} = E_{cd} - E_{ab} = 120\angle 0 - 120\angle 60°$

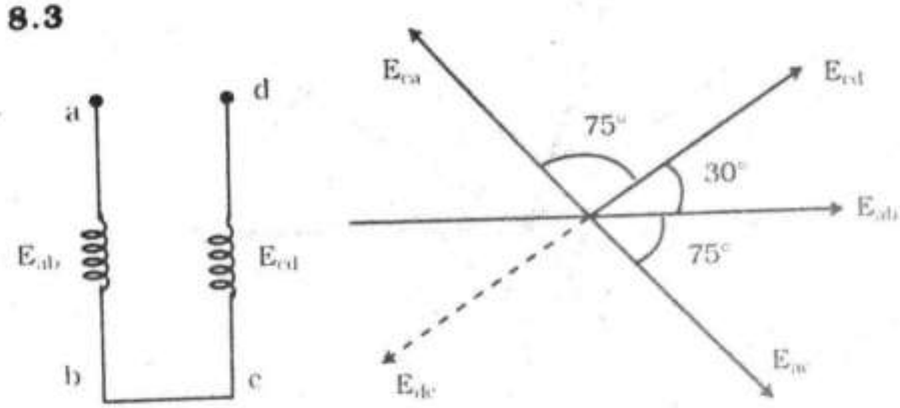
$= 120 + j0 - 60 - j103.923 = 60 - j103.923 = 120\angle -60°$

(b) $E_{ac} = E_{ab} + E_{dc} = E_{ab} - E_{cd}$

$= 120\angle 60° - 120\angle 0° = 60 + j103.923 - 120$

$= -60 + j103.923 = 120\angle 120°$

8.3



$\bar{E}_{ac} = \bar{E}_{ab} + \bar{E}_{dc} = 100\angle 0 + 100\angle -150°$

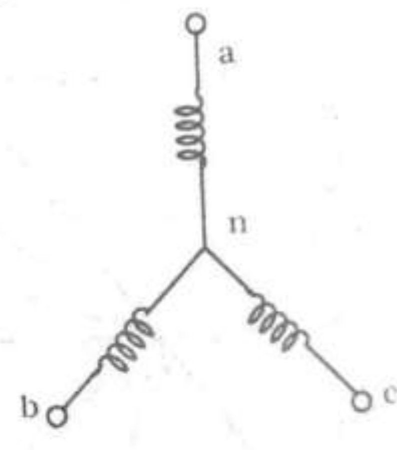
$= 100 + j0 + 86.6 - j50$

$= 13.397 - j50$

$= 51.76\angle -75°$

$\bar{E}_{an} = 51.76\angle 105°$

8.4 From oscillogram 1, phase sequence : a – b – c



$V_{ab/max} = 141$ volts $= \sqrt{3}$ (max. voltage per phase)

$V_m = \dfrac{141}{\sqrt{3}} = 81.406$ Volts

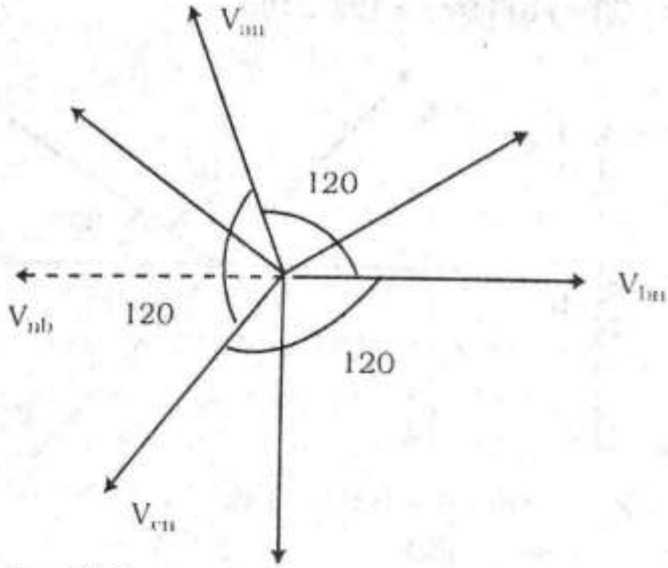
$V_m = 81.406$ Volts (magnitude)

$\therefore \quad V_{rms} = \frac{V_m}{\sqrt{2}} = 57.6$ Volts

$V_{ab} = V_{bn} - V_{an} = 57.6$ Volts (magnitude only)

Phase voltage sequence : an – bn – cn

8.5



8.6 n = 6, I_p = 100 amp

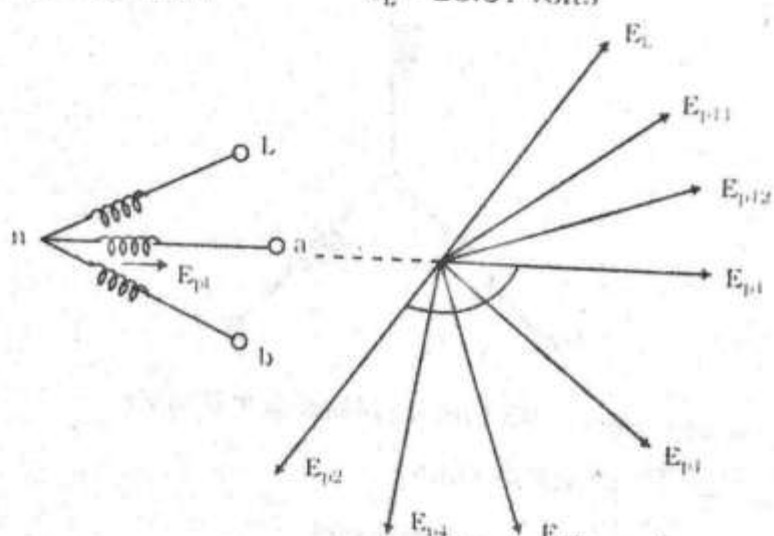
$I_1 \quad = 2I_p \sin \frac{180°}{n} = 2 \times 100 \times \sin \frac{180}{6} = 100$ amp

$\therefore \quad I_p \quad = I_1 = 100$ amp

8.7

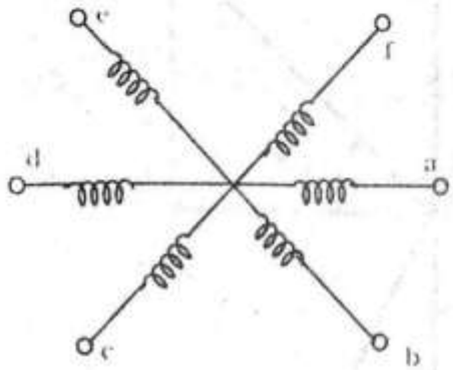
$E_{ab} = E_{an} + E_{nb}$

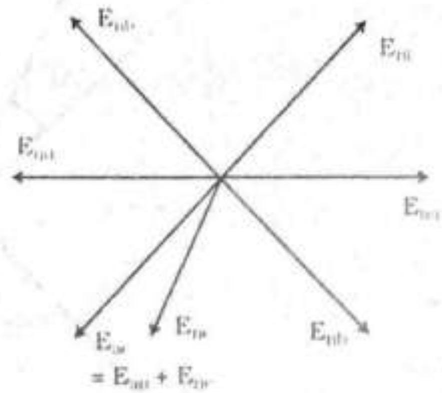
$\therefore \quad$ 1 cm = 50 volts $\qquad E_L = 25.61$ volts



2.1 cm = 25.61 volts $\qquad E_L = 2 \times 50 \times \sin\frac{180}{12} = 25.08$ volts

$B_{ab} = 25.61 \angle -105$ volts





$E_{na} = E_{nb} = E_{nc} = E_{nd} = E_{ne} = E_{nf} = E_p = 132.8$ volts

1 cm = 33.2 volts

$\frac{360}{6} = 60$

$E_{ae} = 6.9$ cm = 230 volts

$$E_{ae} = 2E_p \sin\frac{360}{n}$$

$$= 2 \times 132.8 \times \sin\frac{360}{6}$$

$$= 230 \text{ volts}$$

$V_L = 2300$ V. $V_p = 2300/\sqrt{3} = 1327.9$ Volts

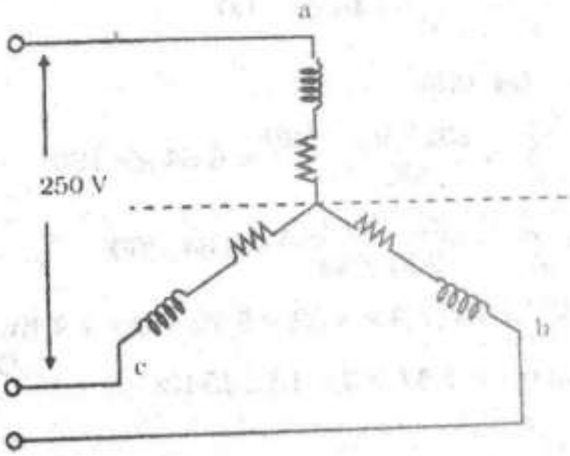
$Z_p = 100 + j173.2 = 200 \angle 60^\circ$

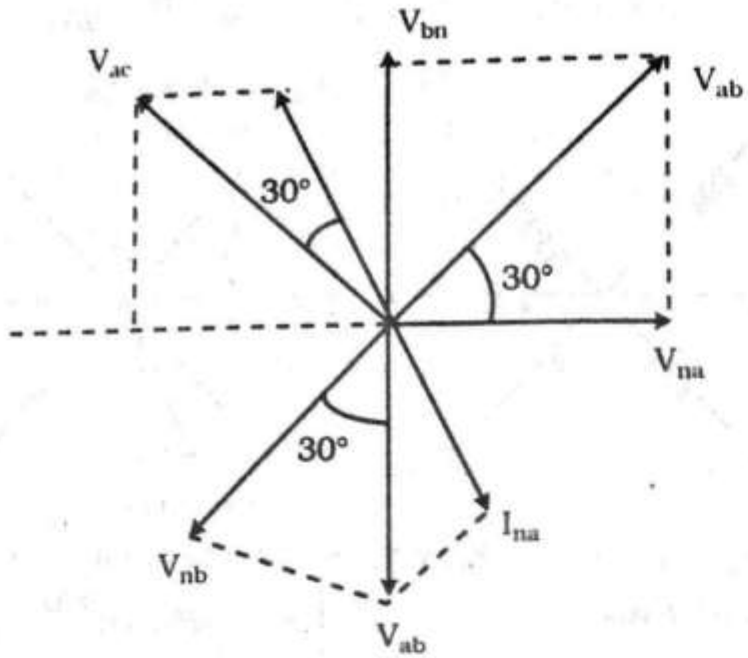
$\therefore \quad I_p = \frac{1327.9}{200 \angle 60} = 6.634 \angle -60^\circ$ amp

$I_L = 6.64$ amp

Power phase = $(6.64)^2 \times 100 = 5.5$Kw

$P_t = 3 \times P_{phase} = 3 \times 4.4 = 13.22$ Kw





By vector—

V_{na} = 1327.7 ∠0 = 1327.9 + j0

V_{nb} = 1327.9 ∠– 120 = 663.96 – j1150

V_{nc} = 1327.9 ∠+ 120 = – 669 + j1150

V_{ba} = V_{bn} + V_{na} = – V_{nb} + V_{na}

= + 603.96 + 1150 + 1327.9 + j0

= 1991.87 + j1150 = 2300 ∠30°

$$I_{an} = \frac{V_{na}}{Z_{na}} = \frac{1327.9\angle 0}{100\angle 60^\circ} = 46.64\angle 60^\circ$$

I_1 = I_{na} = 6.64 amp

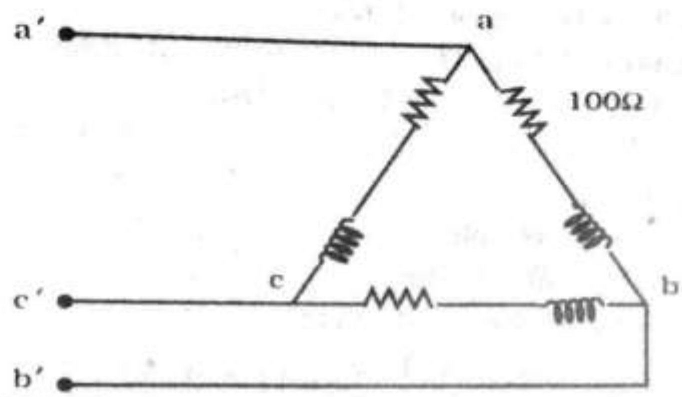
$$I_L = I_{nb} = \frac{V_{nb}}{Z} = \frac{1327.9\angle -120}{200\angle 60^\circ} = 6.64\angle -180^\circ$$

$$I_L = I_{nc} = \frac{V_{nc}}{Z} = \frac{1327.9\angle 120}{200\angle 60^\circ} = 6.64\angle 60^\circ$$

P_{na} = Vi + V′i′ = 1327.9 × 3.32 + 5.75 × 0 = 4.4 Kw

∴ Total power = 4.41 × 3 = 13.225 Kw

8.10



$V_{ba} = 2300 \angle 0$

$V_{cb} = 2300 \angle -120$

$= 1150 - j1991.80$

$V_{ac} = 2300 \angle -240$

$= 1156 + j1991.8$

$I_{ba} = \dfrac{2300 \angle 0^\circ}{200 \angle 60} = 11.5 \angle -60 = 5.75 - j9.96$

$I_{ab} = \dfrac{V_{ab}}{Z} = \dfrac{2300 \angle -120}{200 \angle 60} = 11.5 \angle -180 = -11.5$

$I_{ac} = \dfrac{2300 \angle -240}{200 \angle 60} = 11.5 \angle -300 = 575 + j9.96$

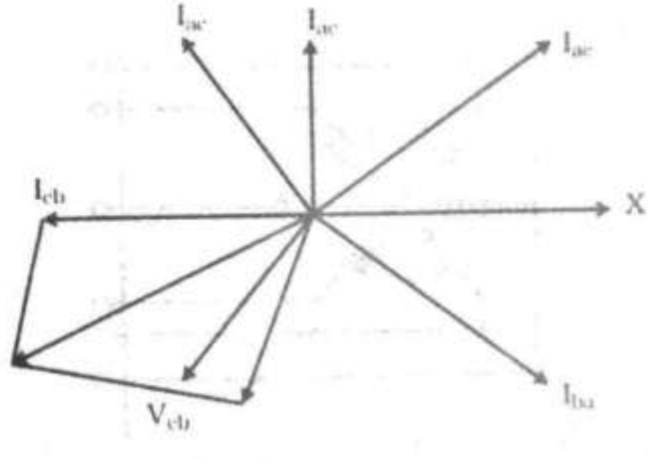
$\therefore \quad |I_p| = 11.44,\ I_L = \sqrt{3} \times 11.5 = 19.92$ amp

$I_{b'b} = I_{bc} + I_{ba} = -I_{cb} + I_{ba} = 11.5 - j0 + 5.75 - j9.96$

$= 19.92 \angle -30^\circ$ amp

$I_{a'a} = I_{cb} + I_{ca} = I_{cb} - I_{ac} = -11.5 + j6 - 5.75 - j9.96$

$= -17.25 - j9.9 = 19.92 \angle -150$



$P_{ba} = |V_{ba}|\ |I_{ba}| \cos(\theta_2 - \theta_i)$

$= 2300 \times 11.5 \cos\{0 - (-60)\}$

$= 2300 \times 11.5 \cos 60^\circ = 13225$ watt $= 13.23$ Kw

P_t of three phase $= 13.23 \times 3 = 39.69$ Kw

8.11

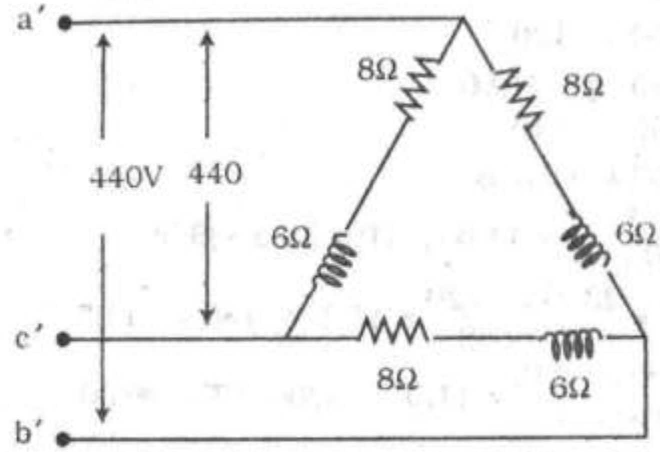
$VA_p = V_p I_p$

$V_L = 440 \angle 0^\circ$ volt

Let $V_{ab} = 440 \angle 0$ volts

$Z_p = 8 + j6$ ohms $= 10 \angle 36.87^\circ$

$\therefore \quad I_p = I_{ab} = \dfrac{V_{ab}}{Z_{ab}} = \dfrac{440 \angle 0}{10 \angle 36.87} = 44 \angle -36.87^\circ$



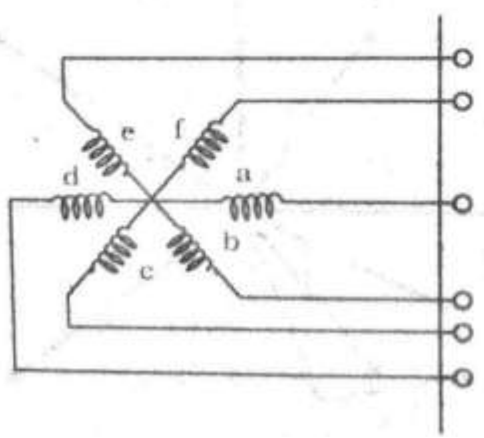
$VA_p = 440 \times 44 = 19360$ r.f $= \sin \theta_p = \sin 36.87 = 0.6$

$rVA_p = 440 \times 44 \times 0.6 = 11616$

$VA_t = VA_p = 58080$. $vV_{at} = rVA_t = 3 \times 11616 = 34848$

r.f of the system remains the same.

8.14



$V_{an} + V_{nb} = 156$ volts

$2V_p = 156$ volts

$V = 78$ volts

$V_L = 2V_p \times \sin \frac{180^\circ}{n}$

$= 2 \times 78 \sin 30 = 70$ volts

8.15

$E_L = 2E_p \sin \frac{180}{n}$ or, $100 = 2F_p \sin \frac{180}{12}$

$E_p = \frac{100}{2 \times \sin \frac{180}{12}} = 193.18$ volts

$E_E = 2E_p \sin \frac{360}{11} = 2 \times 193 \sin \frac{180}{6} = 193.185$

Greatest voltage $= 2E_p = 2 \times 193.18 = 386.37$ vots

8.16

$I_L = 2I_p \sin \frac{180}{n}$

$10 = 2I_p \sin \frac{180}{6}$

$\therefore \quad I_p = 10$

$I_L = 2I_p \sin \frac{180}{12}$

$10 = 2 \times I_p \sin 15^\circ$

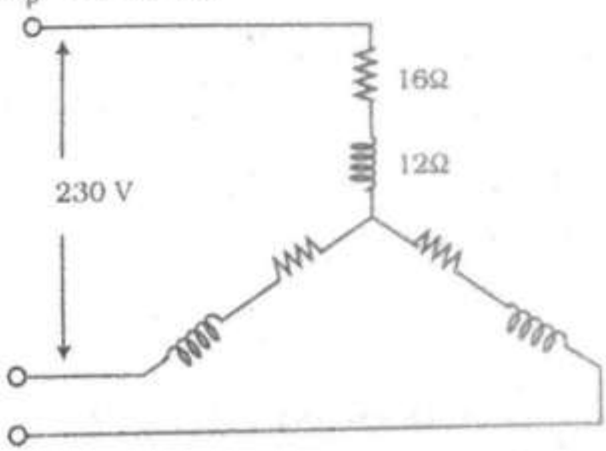
$\therefore \quad I_p = 19.3$ amp

8.21

$Z_p = 16 + j12 = 20 \angle 36.87^\circ$

$V_p = \frac{V_L}{\sqrt{3}} = \frac{230}{\sqrt{3}} = 132.73$ volts

$\therefore \quad I_p = \frac{V_p}{Z_p} = \frac{230}{20 \angle 30.87}$



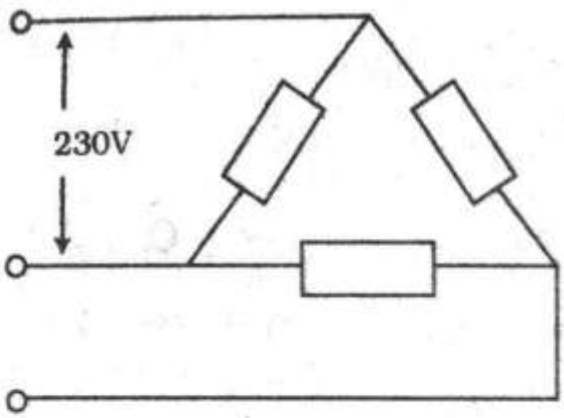
For balanced wye load.

$I_L \;= I_p = 6.64 \angle -36.87A$

Total power.

$P_t \;= 3\, V_p I_p \cos\theta_p$

$\;\;= 2116.14$ watts



For Delta Connection :

Here.

$V_L \;= V_p = 230$ V

$I_p \;= \dfrac{V_p}{Z_p} = \dfrac{230}{20\angle 30.87}$

$\therefore\; I_L \;= V_3 I_p = \dfrac{230}{\sqrt{3}} \times 11.5 = 19.9$ A

$P_t \;= \sqrt{3}\, V_L I_L \cos\theta_p = 6348.44$ watts

8.22

$I_L \;= 2I_p \sin\dfrac{180}{n}$, $10 = 2 \times I_p \sin\dfrac{180}{12}$

$I_p \;= 19.32$ amp

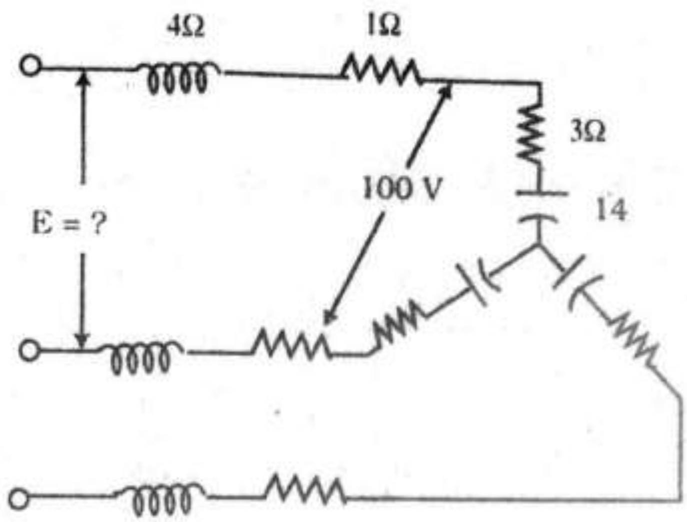
$E_L \;= E_p = I_p Z_p = 19.32 \times 9.43 \angle -58$

$\;\;= 182.18 \angle -58$ volt

8.23

$V_L \;= 100$ V

$I_p \;= \dfrac{V_p}{Z_p} = \dfrac{57.735}{\sqrt{3^2+4^2}} = 11.547$ amp



Now.

$$E_p = I_p\,(j4 + 1 + 3 - j4) = 11.547 \times 4 = 46.108 \text{ volts}$$

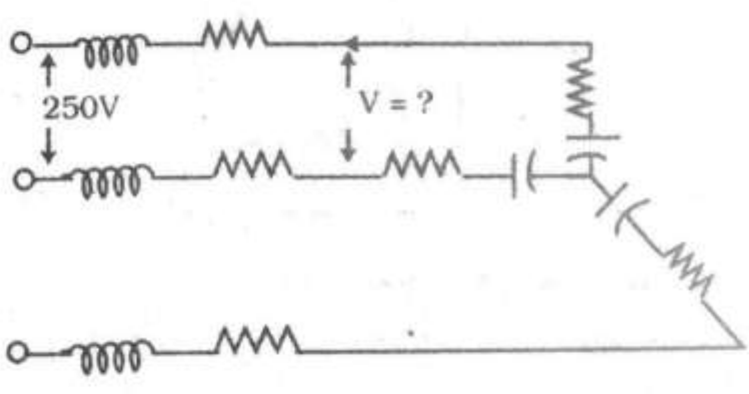
$$\therefore\quad E_L = \sqrt{3}E_p = 80 \text{ volts}$$

8.24

Sending voltage $E_L = 250$ V

$$\therefore\quad E_p = \frac{250}{\sqrt{3}} = 144.39 \text{ Volt}$$

$$\therefore\quad I_p = \frac{E_p}{1 + j2 + 8 + j6} = \frac{144.39}{12.042\ \angle 41.63} = 11\ \angle -41.63^\circ$$



$$\therefore\quad V_p = I_p(8 + j6) = (12\ \angle -41.63)\ (10\ \angle 36.87) = 120\ \angle -4.76$$

$$V_L = \sqrt{3}V_p = 207.85 \text{ volt}$$

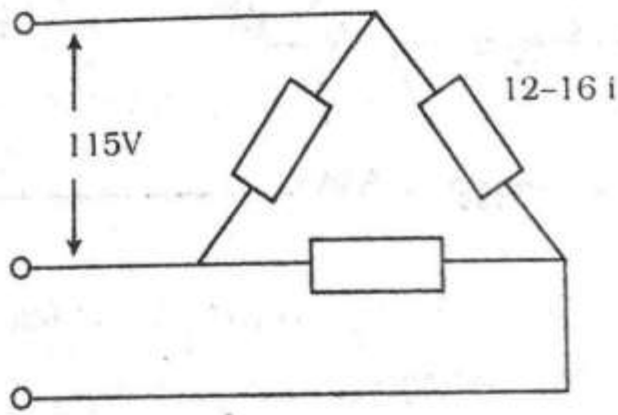
8.25

Here,

$V_p \ = V_l = 115 \text{ V}$

$\therefore \quad I_p \ = \dfrac{V_p}{Z_p} = \dfrac{115}{12 - j16} = 5.73 \angle 53.13^\circ$

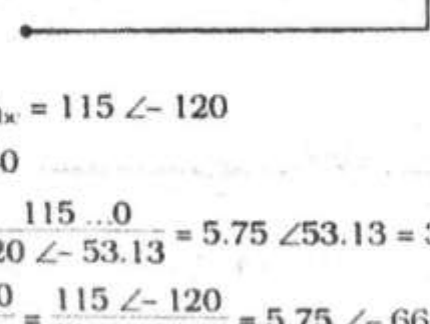
$\therefore \quad I_l \ = \sqrt{3} I_p = 9.96 \angle 53.13^\circ$



$Z_{perphase} = 12 - j16 = 20 \angle -53.13^\circ$

$V_p \ = 115 \text{ volt}$

$|I_p| = \dfrac{115}{20} = 5.75 \text{ amp}$



$V_{ab} \ = 115 \angle 0,\ V_{bc} = 115 \angle -120$

$V_{ca} \ = 115 \angle -240$

$I_{ab} \ = \dfrac{V_{ab} \angle 0}{Z} = \dfrac{115 \ldots 0}{20 \angle -53.13} = 5.75 \angle 53.13 = 3.45 + j4$

$I_{bc} \ = \dfrac{V_{ab} \angle -120}{Z} = \dfrac{115 \angle -120}{20 \angle -53.13} = 5.75 \angle -66.87^\circ = 2{\cdot}26 - j5.29$

$I_{ca} = V_{bc} \angle -120/Z$

$= 115 \angle +120/20 \angle -53.13$

$= 5.75 \angle -186.87$

$= 5.71 + j0.68$

$I_{a'a} = I_{ab} + I_{ac} = I_{ab} - I_{ca} = 3.45 + j4.6 + j5.71 - j0.69$

$= 9.16 + j3.91 = 9.96 \angle 23.41$

$I_{b'b} = I_{ba} + I_{bc} = I_{bc} - I_{ab} = 2.26 - j5.29 - 3.45 + j4.6$

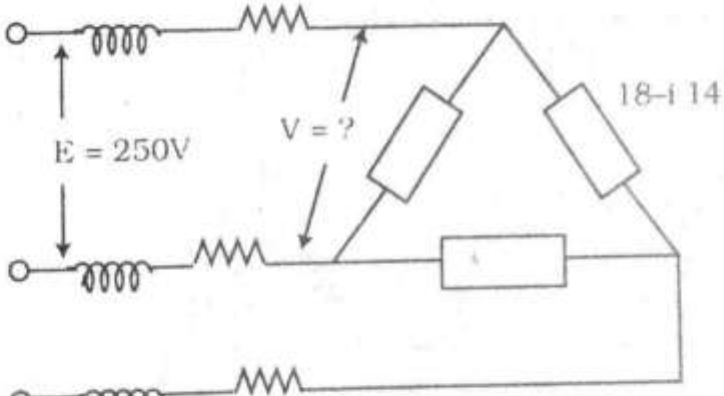
$= 1.19 - j9.89 = 9.96 \angle -96.93$

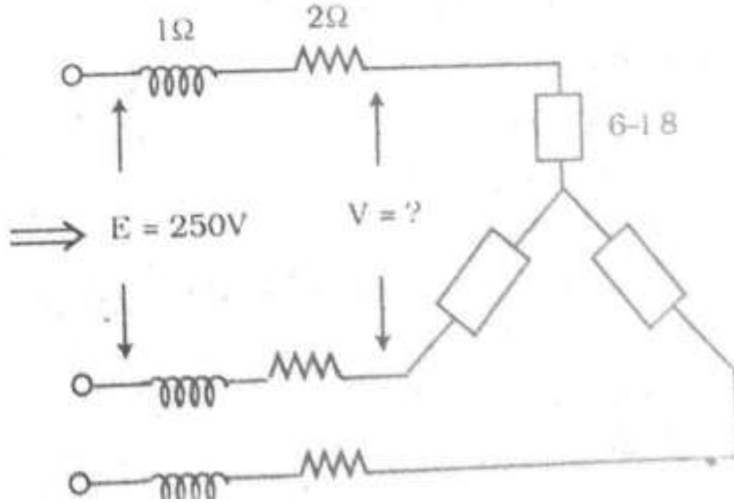
$I_{c'c} = I_{ca} + I_{cb} = I_{ca} - I_{bc} = -5.71 + j0.69 - 2.26 + 5.29$

$= -7.97 + j5 + j5.98 = 9.96 \angle 113.118$

$\therefore$ Line to line (I_{Line}) = 9.95 amp

8.26





Equivalent wye of delta load.

$$Z_p = \frac{18 - j24}{3} = 6 - j8 \text{ ohm}$$

Now, $E_p = \frac{F_L}{\sqrt{3}} = \frac{250}{\sqrt{3}} = 144.34$ volts

$$\therefore \quad I_p = \frac{E_p}{Z_p} = \frac{144.34}{1 + j2 + 6j8} = 15.656 \angle 40.6^\circ$$

$$\therefore \quad V_p = I_p(6 - j8) = 156.56 \angle -12.53^\circ$$

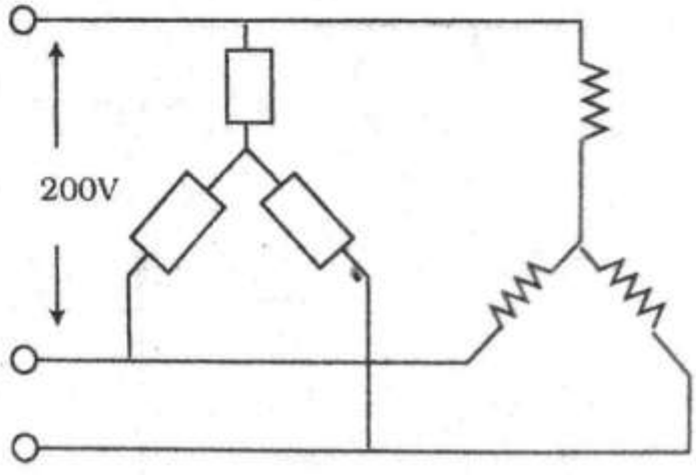
$$V_L = V_3 V_p = 271.17 \angle -12.53^\circ$$

Total power consumed = $3I_p^2R_p = 3(15.656)^2 \times 6 = 4412$ watts

Again $P_t = 3V_pI_p \cos\theta_p$

$$= 3 \times 156.56 \times 15.656 \times \cos\{40.6 - (-12.83)\} = 4412 \text{ watts}$$

8.27



1st Method :

For balanced inductive load, $p = \sqrt{3}\, V_L I_L \cos\theta_p$

or, $5.4 \times 10^3 = V_3 \times 200 \times I_3 \times 0.6$

$\therefore \quad I_{L1} = 25.98$ amp

Since p.f is 0.6 and the load in inductive.

$\therefore \quad I_{L1} = 25.98 \angle -53.13^\circ$

$\therefore \quad \cos\theta = 0.6$

$\theta = 63.13$

For balanced resistive load, $p = \sqrt{3}V_LI_L \cos op$

or, $5 \times 10^3 = \sqrt{3} \times 200 \times I_L^2 \times 1$

Since the load is resistive

$\therefore \quad I_{L2} = 14.43 \angle 0°$

$\therefore$ Resultant current $= 25.98 \angle 53.13 + 14.43 \angle 0$

$= 15.588 - j20.784 + 14.43$

$= 36.5 \angle -34.7°$

$I_L \qquad = 10 \angle 36.87$

$\therefore$ Line drop per phase $V_p = 10 \angle 36.87 (1 + j5)$

$= 51 \angle 115.56°$

$\therefore \quad V_{line} = \sqrt{3} \times 51 \angle 115.56 = 88.33 \angle 115.56°$

$\therefore$ Voltage at the load terminal is $- 230 \angle 0 - 88.33 \angle 115.56$

$= 238 + 38.11 - j79.69 = 279.7 \angle -16.55$ Ans.

$I_L \quad = 14.484$ amp

Since it is star connection,

I_p is the same of I_1 i.e., 14.484 amp

$$X_{cp} = \frac{V_p}{I_p} = \frac{132.8}{14.484} = 9.14 \text{ ohm}$$

$$V_p = \frac{V_L}{\sqrt{3}} = 230/\sqrt{3} = 132.8$$

$$\frac{1}{2\pi fc} = X_{cp} = 9.14$$

$c = 289\ \mu f$

When p.f leading $\sqrt{3}\, X_L I_L = 11.55 \times 10^3$

$$\therefore \quad I_L = \frac{11.55 \times 10^3}{\sqrt{3} \times 236} = 29 \text{ amp}$$

$\therefore \quad I_p = 29$ amp

$$X_{cp} = \frac{V_p}{I_p} = \frac{132.8}{29} = 4.58 \text{ ohm}$$

$$\frac{1}{2\pi fc} = 4.58$$

$$c = \frac{1}{\pi f \times 4.58} = 5.79 \times 10^{-7} = 579.11 \mu f$$

8.32.

For the load, $P = \sqrt{3}V_L I_L \cos\theta = 10 \times 0.5 = 5$ Kw

$\therefore \quad P_X = 10 \sin\theta = 10 \times \sin 60^\circ = 8.66$ K vars

For capacitive bank p = 0

$p_X = x$ (say)

Since the p.f is reduced to 0.866 (lagging) from 0.5 (lagging)

We have $= \tan(\cos^{-1} 0.866) = \dfrac{8.66 - x}{5}$

$\therefore \quad x = 5.773$ K vars

8.33

When they are Delta connected and p.f is 0.866 lagging

$P_X = \sqrt{3}V_L I_L \sin\theta_p$

or, $\quad 5.773 \times 10^3 = \sqrt{3} \times 230 \times I_1 \times \sin(\cos^{-1} 0.866)$

$\therefore \quad I_1 = 28.98 \therefore I_p = \dfrac{28.98}{\sqrt{3}}$

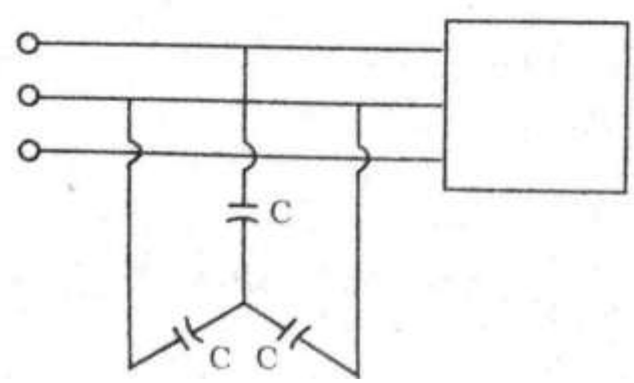
Now, $Z_p = \dfrac{V_p}{I_p} = \dfrac{230}{\dfrac{28.98}{\sqrt{3}}} = 13.746$ ohm

$\therefore \quad Z_p = \dfrac{1}{\omega c}$ or, $c = \dfrac{1}{\omega Z_p} = \dfrac{1}{2\pi \times 60 \times 19.746} = 193.4$

For star

When p.f 0.866 load we got before line to line voltage

$= \sqrt{3} \times 52 = 90.07$ volts



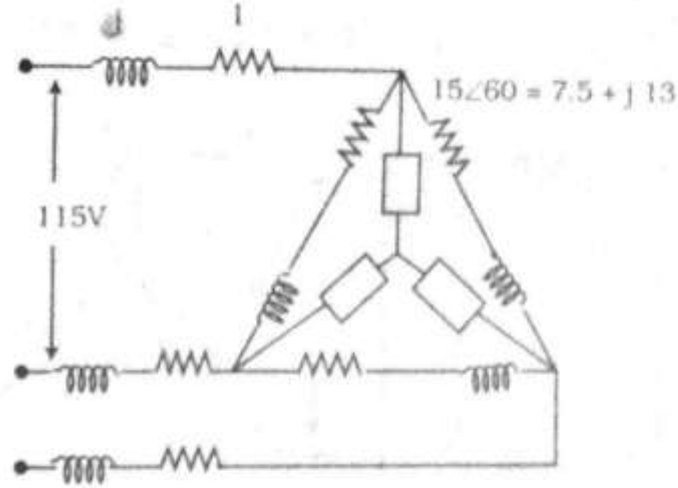
Power loss in the supply lines $= 3I_L^2R_L = 3 \times (10.4)^2 \times 1$

$= 325$ watt

Power dissipated in the load $= 3 \times (10.41)^2 \times 2.5$

$= 812.76$ watts

8.34



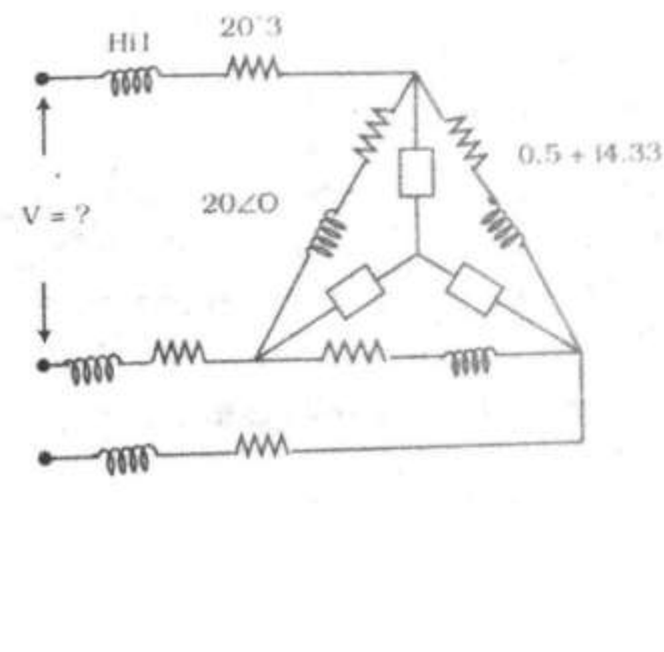
Converting to wye $Z_p = \frac{7.5 + j13}{3} = 2.5 + j4.33$

Now, curent. $I_p = \frac{V_p}{Z_{ep}} = \frac{\frac{115}{\sqrt{3}}}{1 + j + 2.5 + j4.33} = 10.4 \angle -56.7$

Voltage across the load is, $V_p = (10.4 \angle -56.7)(2.5 + j4.38)$

$= 52 \angle 3.3^\circ$

8.35



Phase current in delta = 20A

Line current in = $20\sqrt{3}$A

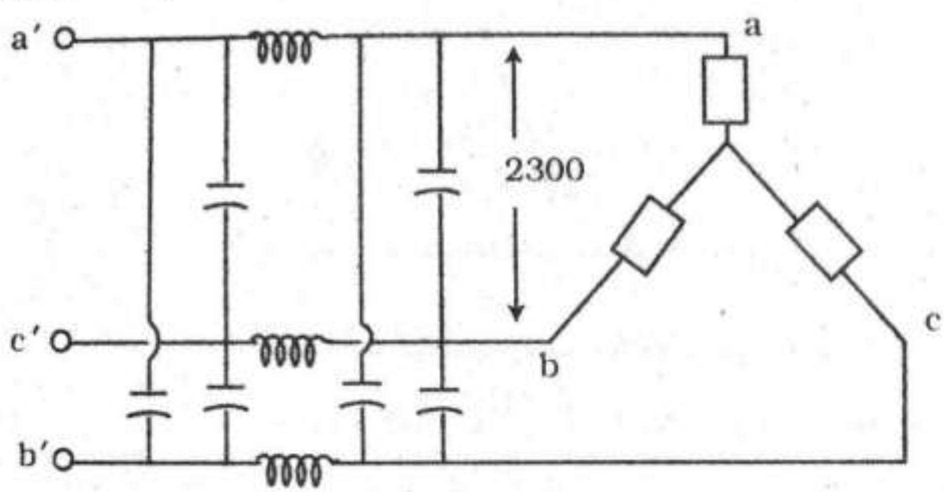
$\therefore$ Impedence in wye = $\frac{15\angle 60}{3} = 5\angle 60 = 2.5 + j4.33$

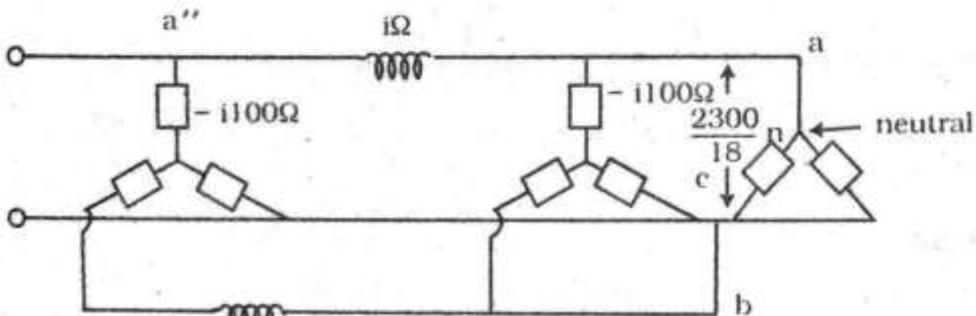
$\therefore$ $V_p = I_pZ = 20\sqrt{3}\,(2.5 + j4.33 + 1 + j1)$

$= 20\sqrt{3} \times 6.376\angle 56.7$

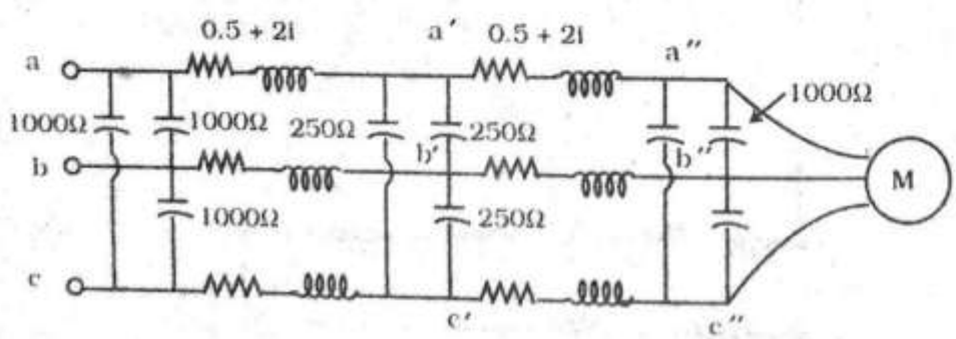
$\therefore$ $V_L = \sqrt{3}X_p = 20 \times \sqrt{3} \times 6.376\angle 56.7 = 382.6\angle 56.7$ volt

8.36





8.37



$$I_L = \frac{P_L}{\sqrt{3}V_L\cos\theta} = \frac{VA}{\sqrt{3}\,V_L} = \frac{120 \times 10^3}{\sqrt{3} \times 2300} = 30.122A$$

$V_{a'n} = V_{a''n} + (0.5 + 2I)\ (30.122 \angle 53.13)$

$= 132.7.91 + (0.5 + 2I)\ (30.122 \angle 53.13)$

$= 1289.65 \angle 2.14168$

$V_{b'n} = V_{b''n} + (2.06 \angle 75.96)\ (30.122 \angle (53.13 - 120)$

$= 1327.91 \angle -120^\circ + (2.06 \angle 75.96)\ [30.122 \angle (53.13 - 120)]$

$= 1289.68 \angle -117.85$

Similarly $V_{c'n} = 1289.68 \angle 122.14$

Again at a, b, c

$V_{an} = V_{a'n} + (0.5 + 2j)\ (30.122 \angle 53.13)$

$= 1289.65 \angle 2.1418 + (0.5 + j)\ (30.122 \angle 53.13)$

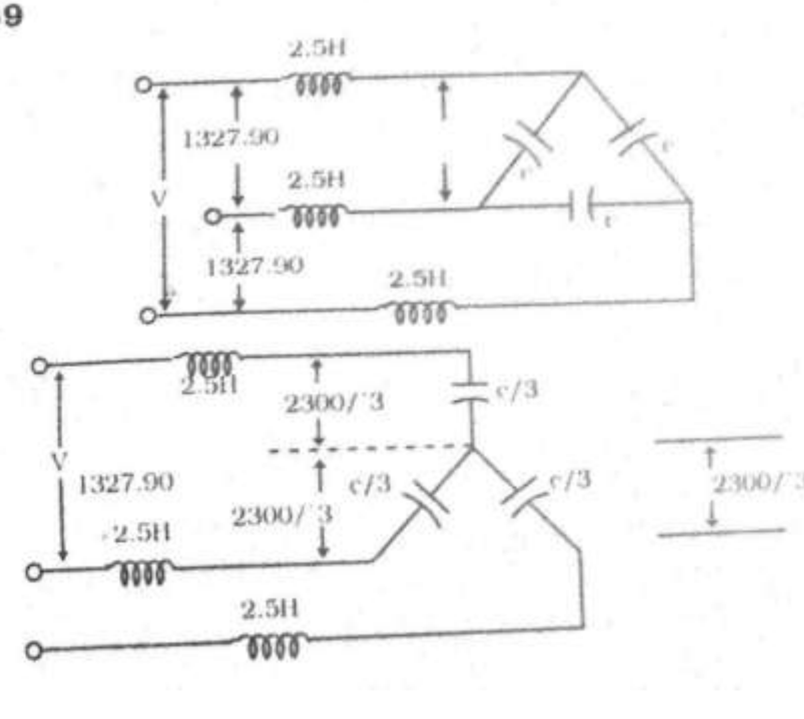
$= 1253.303 \angle 4.41$

$V_{bn} = 1253.303 \angle -115.59$

Similarly. $V_{cn} = 1253.303 \angle 124.41$

8.38 Similar to 37 but take the voltage of a, b, c as input then subtract the voltage drop at (0.5 + 2I)

8.39



For resonance.

$WL = \frac{1}{\omega c}$

$\omega^2 = \frac{1}{LC}$

$F = \frac{1}{2\pi\sqrt{LC}}$

$= \frac{1}{2\pi} \times \frac{1}{\sqrt{2.5 \times 10^{-3} \times 8.683 \times 10^{-6}}}$

$= 341.59$ Hz

$V = 2300/\sqrt{3}$

$I_L = I_{phase}$

$= \frac{10 \times 10^3}{2300/\sqrt{3} \times \sqrt{3}} = 4.347$ A

$\therefore \quad Z_1 = \frac{V}{I_L} = 305.476$

As R_c is negligible.

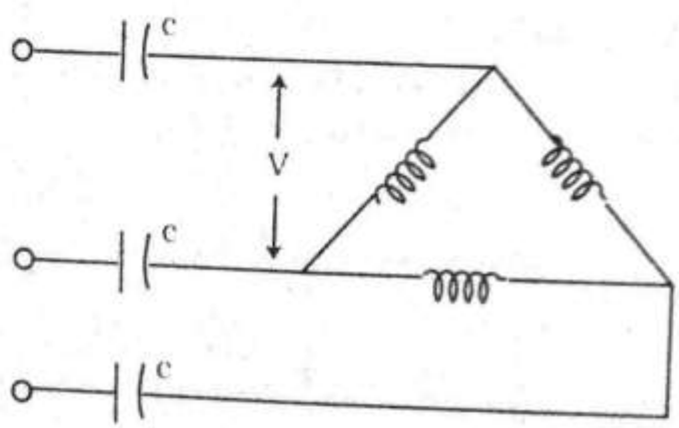
$X_c = 305.4$

$\frac{1}{wc} = 305.476$

$c = \frac{1}{w \times 305.476}$

$= 8.683 \mu f$

8.40



$36 + (100 \times 2\pi \times 800 \times 10^{-3})\, j = 36 + 105.02\, j$

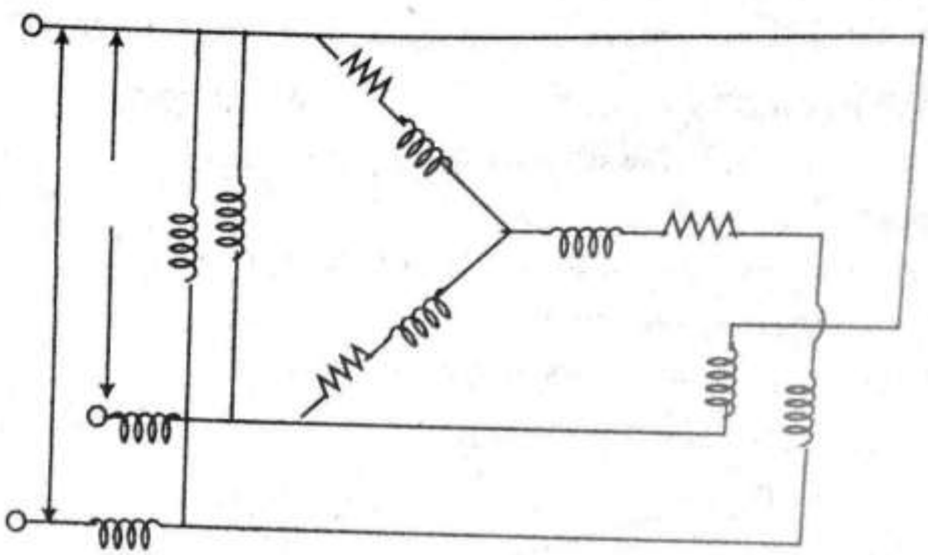
As resonant frequency.

$f = \frac{1}{2\pi\sqrt{LC}}$

$= \frac{1}{2\pi} \times \frac{1}{\sqrt{100 \times 10^{-3} \times c}}$

$\frac{P_X}{W_p} = \sqrt{3}$

8.41.



Phase sequence V_{na}, V_{nc}, Y_{nb} (a, c, b) X_{ac}, V_{cb}, V_{ba}—

$V_{ac} = 100 \angle 0$

$V_{cb} = 100 \angle -120$

$V_{ba} = 100 \angle 120°$

$W_a = V_{ac}\, I_{an} \cos \theta \Big]_{V_{ac}}^{I_{an}}$

$W_b = V_{bc} I_{bn} \cos \theta \Big]_{V_{bc}}^{I_{bn}}$

$$V_p = \frac{100}{\sqrt{3}} = 57.74 \text{ V}$$

$V_{na} = 57.74 \angle 0$. vots

$V_{nc} = 57.74 \angle -120°$

$V_{nb} = 57.74 \angle 120°$

$V_{ac} = V_{an} + V_{nc}$

$= V_{nc} - V_{na}$

$= 57.74 \angle -120° - 57.74 \angle 0$

$= -28.87 - j50 - 57.74$

$= 100 \angle -150°$

$Z_p = 3 + j10 = 10.44 \angle 73.3°$

$I_p = V_p/Z_p = 57.74/10.44 \angle 73.3 = 5.53 \angle -73.3$

$W_a = (100 \angle -150) \times (5.53) \cos(-103.3) = 127.22$ watt

$W_b = 100 \times 5.53 \cos 43.3 = 402.46$ watt

Power in each phase $= (W_a + W_b)/3 = 1.75$ watt

8.42.

Power. $w_a + w_b = 5000$ reactive power $= \sqrt{3}(w_a - w_b) = 20{,}000$

or. $w_a - 5000 + w_a = 20000/\sqrt{3}$ $\therefore$ $w_a = 8273.5$ watt

$w_b = -3273.5$ watt

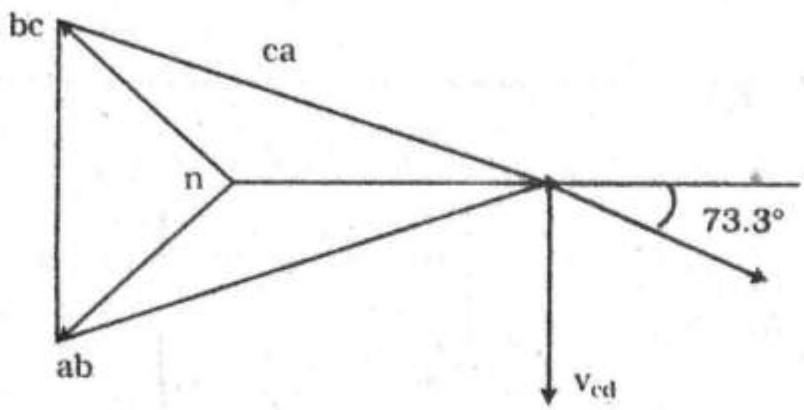
8.43

The reading of W_R in $W_R = V_{cb}I_{na}\cos(V_{cb}I_{na})$

$= V_LI_L\cos(90 - 73.3)$

$= V_LI_L\sin 73.3 = 100 \times 5.53\sin 73.3$

$= 529$ Watts



Total reactive volt – amp taken by the load is—

$P_x = \sqrt{3}V_LI_L\sin\theta = 917.43$ vars

$\therefore \dfrac{P_X}{W_R} = \dfrac{\sqrt{3}V_LI_L\sin 73.3}{V_LI_L\sin 73.3} = \sqrt{3}$

8.44

If phase angle is θ then, $W_R = V_LI_L\sin\theta$

$P_X = \sqrt{3}V_LI_L\sin\theta$

50.51 is similar to number 49.

8.45.

$W_a = V_LI_L\cos(\theta - 30)$

$w_b = V_LI_L\cos(\theta + 30°)$

$\therefore \quad W_a + W_b = V_L I_L \{\cos(\theta - 30°) + \cos(\theta + 30°)\}$

$= \sqrt{3}\, V_L I_L \cos\theta$

$W_a - W_b = V_L I_L \{\cos(\theta - 30) - \cos(\theta + 30°)\}$

$= V_L I_L \sin\theta$

8.46.

$$\tan\theta = \frac{\sqrt{3}(1200 + 400)}{1200 - 400}$$

$\therefore \quad \theta = 73.9° \; p.f = \cos\theta = 0.277$

Power $Z = W_a + W_b = 800$ watt Now. $p = \sqrt{3}\, V_L I_L \cos\theta$

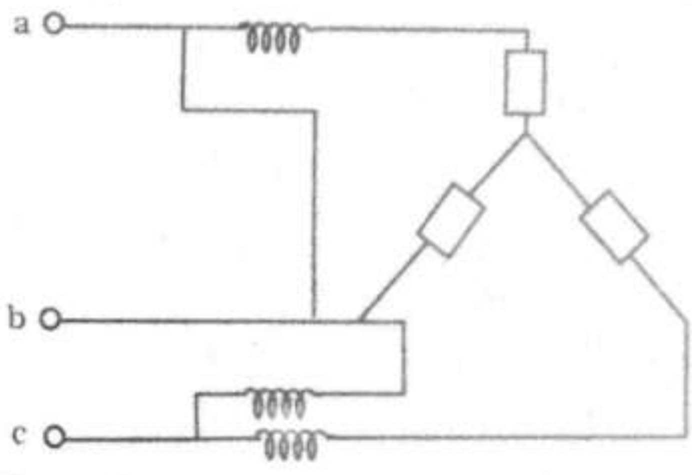
or. $P \times \cos\theta \;\therefore P_X = \dfrac{P}{\cos\theta} = 2888$ VA

8.47.

$W_a = 1000$ Watts

$W_b = 400$ Watts

$P = 1400$ watts



$P_X = \sqrt{3}\,(W_a - W_b) = 1039.28$ vars

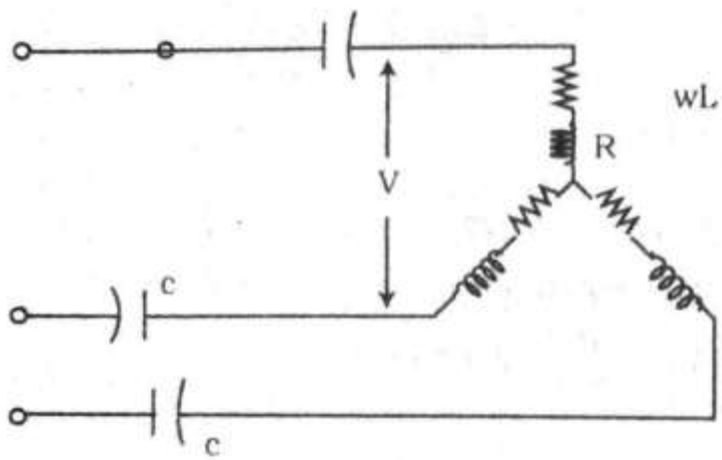
$$p.f = \frac{P}{\sqrt{P^2 + P_X^2}} = 0.802$$

$\therefore \quad W_a = \sqrt{3}\, V_L I_L \cos(\theta - 30°),\; W_b = \sqrt{3}\, V_L I_L \cos(\theta + 30)$

$\therefore$ The voltage sequence is ACB.

$$800^2 = \left(\frac{1}{2_\pi}\right)^2 \times \frac{1}{\frac{100}{3} \times 10^{-3} \times C}$$

$$C = 2.968 \times 10^{-7} f$$



$wL \} r + iw$

$$= \frac{36 + 105.02\, i}{3}$$

$$= 12 + 35\, i$$

(b) Do yourself (Hints. parallel resonance)

[Previous page 41. 42]

8.48

$V_l = 51.76$ volts

$$V_L = V_p \sin \frac{180}{n}$$

or. $51.76 = V_p \sin \dfrac{180}{12}$

$\therefore \quad V_p = 100$ V

$Z_p = 5 + j4$

$$\therefore \quad I_p = \frac{V_p}{Z_p} = \frac{100}{5\,\angle 53.13} = 20\,\angle{-53.13^\circ}$$

Power $= V_p I_p \cos\theta = 12 \times 100 \times 20 \times \cos(53.13) = 14400$ watts

$pf = \cos 53.13 = 0.6$

$I_p = I_L = 20$ amp

8.49

$e_{na} = 127 \sin \omega t + 50 \sin (3\omega t - 30°) + 30 \sin (5\omega t + 40°)$

$e_{nb} = 127 \sin (\omega t - 120°) + 50 \sin (3\omega t - 30 - 120°) + 30 \sin (5\omega t + 40 - 120°)$

$e_{nc} = 127 \sin (\omega t - 240) + 50 \sin (3\omega t - 30 - 240°) + 30 \sin (5\omega t + 40 - 240)$

$\therefore \quad e_{ab} = e_{na} - e_{nb}$

$= [127 \sin \omega t + 50 \sin (3\omega t - 30°) + 30 \sin (5\,\omega t + 40)] -$

$[127 \sin (\omega t - 120°)$

$+ 50 \sin (3\omega t - 30 - 120) + 30 \sin (5\,\omega t + 40 - 120)]$

= Do the rest of the math yourself.

Chapter–9

Unbalanced polyphase circuits

9.1

$Z_{an} = 10 + j0$

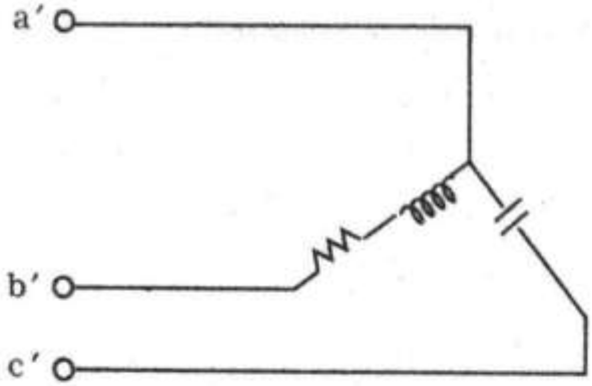
$Z_{bn} = 10 + j10$

$Z_{cn} = 0 - j20$

$I_{an} = I_{a'a} = 3.66 \angle 15$ amp

$I_{bn} = I_{b'b} = 14.56 \angle -125.1$ amp

$I_{cn} = I_{c'c} = 11.98 \angle 66.7$ amp



$\therefore\ V_{an} = I_{an} Z_{an} = (3.66 \angle 15)(10 \angle 0) = 36.6 \angle 15$ volts

$V_{bn} = I_{bn} Z_{bn} = (14.56 \angle -125.1)(19.192 \angle 45^\circ) = 205.9 \angle -80^\circ$

$V_{cn} = I_{cn} Z_{cn} = (11.98 \angle 66.2)(20 \angle -90^\circ) = 239.6 \angle 23.8$ volts

9.2 $P_{an} = V_{an} I_{an} \cos\theta \Big]^{V_{an}}_{I_{an}} = 36.6 + 3.66 \times \cos(15 - 15)$

$= 134$ watts

$P_{bn} = 205.9 \times 14.56 \cos(-80.1 + 105.1) = 2119.84$ watts

$P_{cn} = 239.6 \times 11.98 \cos(23.8 - 66.2) = 0$ watts

9.4 $\dfrac{E_{n'a} - I_{n'n} Z_n}{Z_a} + \dfrac{E_{n'b'} - I_{n'n'} Z_n}{Z_b} + \dfrac{E_{n'c'} - I_{nn'} Z_n}{Z_c} = I_{nn'}$

or, $E_{n'a} Z_b Z_c + E_{n'b'} Z_c Z_a + F_{n'c'} Z_a Z_b = I_{nn'} Z_n (Z_b Z_c + Z_c Z_a + Z_a Z_b)$

$\therefore\ I_{nn'} = \dfrac{E_{n'a} Z_b Z_c + E_{n'b'} Z_c Z_a + E_{n'c'} Z_a Z_b}{Z_a Z_b Z_c + Z_n (Z_b Z_c + Z_c Z_a + Z_a Z_b)}$

9.7 Soln :

$$W_{ba} = V_{ba} I_{b'b} \cos\theta \Big]_{I_{b'b}}^{V_{ba}}$$

$$= 200 \times 35.06 \cos(180 - 145)$$

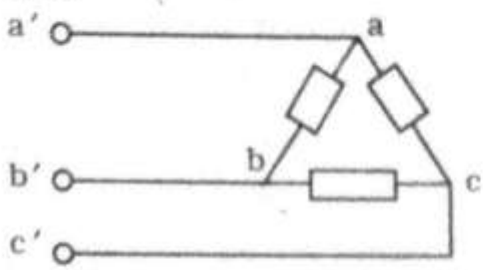
$$= 5742.25$$

$$V_{ba} = V_{ab}\angle 80^\circ$$

$$V_{b'b} = 37.55 \angle 45.55^\circ$$

$$W_{ca} = 200 \times 21.66 \cos\{120 - (-126.75)\} = 4301.97$$

9.11



$$Z_{ab} = 10\angle -60^\circ,\ Z_{bc} = 5\angle 0^\circ$$

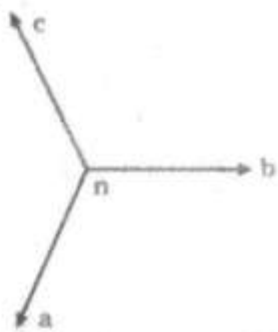
$$Z_{ca} = 10\angle 60^\circ$$

Since the three phase supply voltage are balanced and the phase sequence is cb – ba – ac.

$$\therefore\ V_{cb} = 100 \angle 0$$

$$V_{ba} = 100 \angle -120$$

$$V_{ac} = 100 \angle 120^\circ$$



$$Z = 100 - j100 = 141.4\angle -45^\circ$$

[3φ sequence is always connected in the clockwise direction.]

Now,

$$I_{ba} = \frac{V_{ba}}{Z_{ab}}$$ [Since the impedence are biletaral $Z_{an} = Z_{bu}$]

$$= \frac{100 \angle 120}{10 \angle -10^\circ} = 10 \angle -60 \text{ amp}$$

$$I_{cb} = \frac{V_{cb}}{Z_{cb}} = \frac{100 \angle 0}{5 \angle 0} = 20 \angle 0^\circ$$

$$I_{ac} = \frac{V_{ac}}{Z_{ca}} = \frac{100 \angle 120}{10 \angle 60^\circ} = 10 \angle 60^\circ$$

Now,

$$I_{a'a} = I_{ac} + I_{ab} = I_{ac} - I_{ba} = 10 \angle 60 - 10 \angle -60$$

$$= 8 + j8.66 - 8 + j8.66$$

$$= 17.32 \angle 90^\circ$$

$$I_{b'b} = I_{ba} - I_{ab} = 10 \angle -60 - 20 \angle 0^\circ = 5 - j8.66 - 20 + j0$$

$$= 17.32 \angle 150$$

$$I_{c'c} = I_{cb} - I_{ac} = 20 \angle 0 - 10 \angle 60 = 20 - 5 - j8.66$$

$$= 17.32 \angle -30^\circ$$

For opposite sequence the sequence will be cb – ac – ba

$$\therefore \; V_{cb} = 100 \angle 0 \; V_{ac} = 100 \angle -120^\circ \; V_{ba} = 100 \angle 120^\circ$$

$$\therefore \; I_{ba} = \frac{V_{ba}}{Z_{ab}} = \frac{100 \angle 120^\circ}{10 \angle -60} = 10 \angle 180^\circ$$

$$\therefore \; I_{cb} = \frac{V_{cb}}{Z_{bc}} = \frac{100 \angle 0}{5 \angle 0} = 20 \angle 0$$

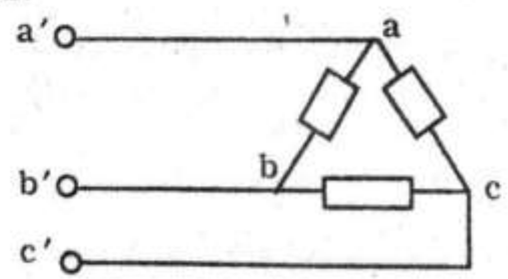
$$I_{ac} = \frac{V_{ac}}{Z_{ca}} = \frac{100 \angle -120}{10 \angle 60} = 10 \angle -180^\circ$$

$$\therefore \; I_{a'a} = I_{ac} - I_{ba} = 10 \angle -180 - 10 \angle 180^\circ = 10 + 10 = 0$$

$$I_{b'b} = I_{ba} - I_{cb} = 10 \angle 180 - 20 \angle 0 = -10 - 20 = 30 \angle 180^\circ$$

$$I_{c'c} = I_{cb} - I_{ac} = 20 \angle 0 - 10 \angle -180^\circ = 20 + 10 = 30 \angle 0^\circ$$

912.



$$Z_{ab} = 5 \angle 40^\circ$$

$$Z_{bc} = 10 \angle -30^\circ$$

$$Z_{ca} = 8 \angle 45^\circ$$

Since the phase sequence is cb – ac – ba and the line voltage are balanced.

$$\therefore \; V_{cb} = 115 \angle 0, \; V_{ac} = 115 \angle -120 \; V_{ba} = 115 \angle 120$$

Now, $I_{cb} = \frac{V_{cb}}{Z_{bc}} = \frac{115 \angle 0}{10 \angle -30} = 11.5 \angle 30$

$$\dot{I}_{ac} = \frac{V_{ac}}{Z_{ca}} = \frac{115 \angle -120^\circ}{8 \angle 45^\circ} = 14.375 \angle -165^\circ$$

$$I_{ba} = \frac{V_{ba}}{Z_{ab}} = \frac{115 \angle 120^\circ}{5 \angle 40} = 23 \angle 80^\circ$$

Since the currents are leaving the terminals.

$$\therefore\ I_{aa'} = I_{ba} + I_{ca} = I_{ba} - I_{ac} = 23 \angle 80^\circ - 14.375 \angle -165$$

$$= 4 + j22.65 + 13.885 + j3.72$$

$$= 31.863 \angle 55.85^\circ A$$

$$I_{bb'} = I_{cn} - I_{ba} = 11.5 \angle 30 - 23 \angle 80^\circ = 9.96 + j5.751 - 4 - j22.65$$

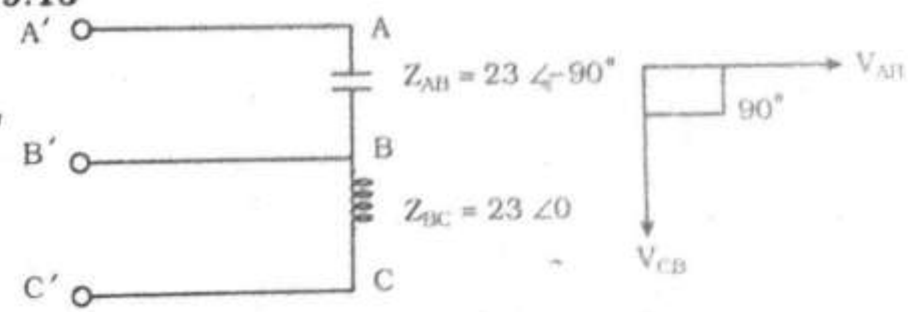
$$= 17.92 \angle -70.57^\circ$$

$$I_{cc'} = I_{ac} - I_{cb} = 14.375 \angle -165 - 11.5 \angle 30^\circ$$

$$= -13.885 - j3.72 - 9.96 - j5.75$$

$$= 25.66 \angle -158.34^\circ$$

9.13



Since the voltage sequence is AB – CB

$$\therefore\ V_{AB} = 115 \angle 0\ V_{CB} = 115 \angle -180^\circ - (-90^\circ)$$

$$= 115 \angle -90^\circ$$

$$\therefore\ I_{AB} = \frac{V_{AB}}{Z_{AB}} = \frac{115 \angle 0^\circ}{23 \angle -90^\circ} = 5 \angle 90^\circ$$

$$I_{CB} = \frac{Z_{CB}}{Z_{BC}} = \frac{115 \angle -90^\circ}{23 \angle 0} = 5 \angle -90^\circ \text{ amp}$$

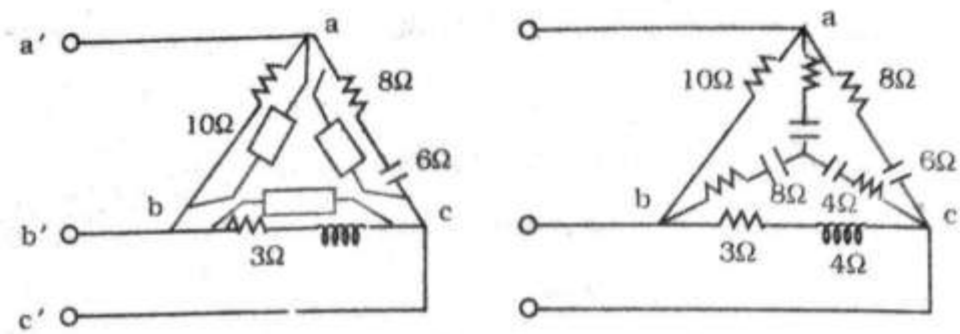
$$I_{BB'} = I_{AB} + I_{CB} = 5 \angle 90^\circ + 5 \angle -90^\circ = 0 + j50 + 0 - 5j$$

$$= 0 \text{ Ans.}$$

9.15

Every impedence of Y is $4 - 3j = 5 \angle -36.87^\circ$

∴ Equivalent impedence of Y to ∇ is $3 \times 5\angle{-36.87^\circ} = 15\angle{-36.87^\circ}$



$$\text{Now } Z_{ab} = \frac{10 \times 15\angle{-36.87}}{10 + 15\angle{-36.87}} = \frac{150\angle{-36.87}}{23.77\angle{-22.25^\circ}} = 6.31\angle{-14.62}$$

$$Z_{bceq} = \frac{(3 + 4j)(15\angle{-36.87^\circ})}{3 + 4j + 15\angle{-36.87}} = 4.74\angle{34.7^\circ}$$

$$Z_{caeq} = \frac{(8 - j6)(15\angle{-36.87^\circ})}{8 - j6 + 15\angle{-36.87^\circ}} = 6\angle{-36.87^\circ}$$

Let the reference voltage be $V_{b'c'}$

$$\therefore \quad V_{b'c'} = 230\angle 0 \; V_{a'b'} = 230\angle{-120}, \; V_{c'a'} = 230\angle{120^\circ}$$

$$I_{ab} = \frac{V_{a'b'}}{Z_{abeq}} = \frac{230\angle{-120^\circ}}{6.31\angle{-14.62}} = 36.45\angle{-105.38^\circ}$$

$$I_{bc} = \frac{230\angle 0}{4.44\angle{34.7}} = 48.52\angle{-34.7}$$

$$I_{ca} = \frac{230\angle{120}}{6\angle{-36.87^\circ}} = 38.33\angle{156.87^\circ}$$

$$\therefore \quad I_{a'a} = I_{ab} - I_{ca} = 36.45\angle{-105.38} - 38.33\angle{156.87^\circ}$$

$$= -9.67 - j35.14 + 35.25 - j15.06$$

$$= 56.34\angle{-63} \text{ Amp}$$

$$I_{b'b} = I_{bc} - I_{ab} = 39.89 - j27.62 + 9.67 + j35.14$$

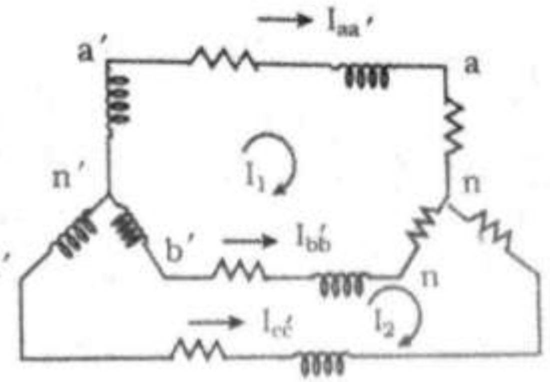
$$= 50.13\angle{8.63^\circ}$$

$$I_{c'c} = I_{ca} - I_{bc} = -35.25 + j15.06 - 39.89 + j27.62$$

$$= 86.42\angle{-150}$$

Hence the meter a'a, b'b, c'c readings will be 56.34 A 50.13 A, 86.42A respectively.

9.6



$$I_1(2 - j + 1 - 3j) - (1 - 3j)\, I_2 = 100 \angle 0 - 100 \angle -120^\circ$$

or, $\quad 5 \angle -53.13\, I_1 - 3.162 \angle -71.5\, I_2 = 173.2 \angle 30^\circ$

$$\Rightarrow (1 - 3j)\, I_1 + (1 - j3 + 3 + 4j)\, I_2$$

$$= 100 \angle -120 - 100 \angle -240^\circ$$

or, $\quad -3.162 \angle -71.57\, I_1 + 4.123 \angle 14.04\, I_2 = 173.2 \angle -90^\circ$

$$I_1 = \frac{\begin{vmatrix} 173.2 \angle 30 & -3.162 \angle -71.57 \\ 173.2 \angle -90 & 4.123 \angle 14.04 \end{vmatrix}}{\begin{vmatrix} 5 \angle -33.13 & -3.162 \angle -71.57 \\ -3.162 \angle -71.57 & 4.123 \angle 14.04 \end{vmatrix}}$$

$$= \frac{513.34 + j496.42 - 519.57 - j173.14}{16 - j13 + 8 + j6}$$

$$= \frac{323.34 \angle 91.1}{25 \angle -16.26} = 12.93 \angle 107.39 \text{ amp}$$

$$I_2 = \frac{\begin{vmatrix} 5 \angle -53.13 & 173.2 \angle 30^\circ \\ -3.162 \angle -71.57 & 173.2 \angle -90 \end{vmatrix}}{25 \angle -16.26}$$

$$= \frac{-692.8 - j519.6 + 409.73 - j363.39}{25 \angle -16.26}$$

$$= \frac{927.25 \angle -107.77}{25 \angle -16.26}$$

$$= 37.09 \angle -91.5$$

Now $I_{a'a} = I_1 = 12.93 \angle 107.36^\circ$ Amp

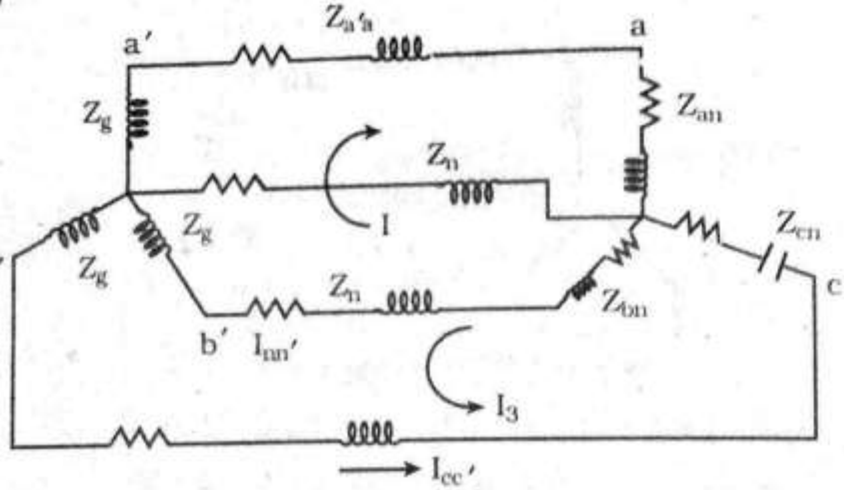
$$I_{b'b} = I_2 - I_1 = 37.09 \angle -91.5 - 12.93 \angle 107.33$$

$$= -0.97 - j37.08 + 3.86 - j12.34$$

$$= 49.5 \angle -35.65^\circ \text{ Amp}$$

$$I_{c'c} = -I_2 = 37.09 \angle -91.5 = 37.09 \angle 99.5$$

9.17



$E_{n'a'} = 1000 + j0 = 1000 \angle 0$

$E_{n'b'} = -500 - j866 = 1000 \angle -120$

$E_{n'c'} = -500 + j866 = 1000 \angle 120$

$Z_9 = 2 + j8 = 8.25 \angle 76°$

$Z_{an} = 20 - 20j$

$Z_{bn} = 50 + j0$

$Z_{cn} = 30 + j52$

$Z_L = 1 + j1$

$Z_n = 2.5 + j$

$Z_9 = 2 + j8$

$$Z_a = Z_9 + Z_9 + Z_{an} = 2 + j8 + 1 + j + 20 - 20j = 23 - j11$$

$$Z_b = Z_9 + Z_9\ Z_{bn} = 2 + j8 + 1 + j + 50 + j0 = 53 + 9j$$

$$Z_c = Z_9 + Z_e + Z_{an} = 2 + j8 + 1 + j + 30 + 52j = 33 + j61$$

$$I_{nn'} = \frac{E_{n'a'}\,Z_bZ_c + E_{n'b'}\,E_cZ_a + E_{n'c'}\,Z_aZ_b}{Z_aZ_bZ_c + Z_n\,(Z_bZ_c + Z_cZ_a + Z_aZ_b)}$$

$$= \frac{100 \angle 0 \times 53.75 \angle 9.64 \times 69.35 \angle 61.59 + 1000 \angle 120 \times 69.35 \angle 61.59 \times 25.5 \angle -25.56 + 100 \angle 120 \times 25.5 \angle -25.56 \times 53.75 \angle 9.64}{25.5 \angle -25.56 \times 53.75 \angle 9.64 \times 69.35 \angle 61.59 + 2.70 \angle 21.8 \times (53.75 \angle 9.64 \times 69.35 \angle 61.58 + 69.35 \angle 61.58 \times 25.5 \angle -25.56 + 25.5 \angle -25.56 \times 53.75 \angle 9.63)}$$

$$= \frac{3727562.3 \angle 71.23 + 1768425 \angle -83.97 + 1370625 \angle 104.08)}{95381.794 \angle 45.67 + 2.70 \angle 21.8\,(3727.5625 \angle 71.22 + 1768.425 \angle 36.02 + 1370.625 \angle -15.93)}$$

$$= \frac{1199417 + j3529322.8 + 185771.59 - j1758640.4 - 33440.78 + j1329447.3}{95381.794 \angle 45.67 + 2.70 \angle 21.8\,(3727.5625 \angle 71.22 + 1768.425 \angle 36.02 + 1370.625 \angle -15.93)}$$

$$= \frac{1051748.5 - j3100129.7}{95381.794 \angle 45.67 + 2.70 \angle 21.8\,(3948.3466 - j4192.8814)}$$

$$= \frac{3273679.7 \angle -71.26}{45381.794 \angle 45.67 + 15550.147 \angle -27.92}$$

$$= \frac{3273679.7 \angle -71.26}{46565.202 + j25910.73}$$

$$= \frac{3273679.7 \angle -71.26}{53288.685 \angle 29.093} = 61.43 \angle -42.167$$

$$\therefore I_{nn'} = 61.43 \angle -42.167$$

$$I_{nn'} = \frac{I_{n'a'} - I_{nn'} Z_n}{Z_a} = \frac{1000 \angle 0 - 61.43 \angle -42.167 \times 2.70 \angle 21.8}{25.5 \angle -25.56}$$

$$= \frac{100 + j0 - 165.861 \angle 20.367}{25.5 \angle -25.56}$$

$$= \frac{100 + j0 - 155.4918 + j57.73}{25.5 \angle -25.56}$$

$$= \frac{846.47907 \angle 3.9106162}{25.5 \angle -25.56}$$

$$= 33.2 \angle 29.471$$

$$I_{b'b} = \frac{E_{n'b'} - I_{nn'} Z_n}{Z_b}$$

$$= \frac{-500 - j866 - 61.43 \angle -42.167 \times 2.7 \angle 21.8}{53.75 \angle 9.64}$$

$$= \frac{-500 - j866 - 155.4918 + j57.73}{53.75 \angle 9.64}$$

$$= 19.361 \angle -138.68$$

$$I_{c'c} = \frac{E_{n'c'} - I_{nn'} Z_n}{Z_c} = \frac{-500 + j866 - 155.4918 + j57.73}{69.35 \angle 61.59}$$

$$= 16.33 \angle 63.77$$

$$Z_{cn} = \frac{10 \times 60 \times 10 \angle -60}{20} = 5 \angle 0^\circ = 5 + j0$$

Voltage sequence is b′a′ – a′c′ – c′b′

$$\therefore \quad E_{b'a'} = 100 \angle 0$$

$$E_{a'c'} = 100 \angle -120$$

$$E_{c'b'} = 100 \angle 120$$

From Fig. 2.

$$(0.5 + j0.5 + 2.5 - j4.33 + 2.5 + j4.33 + 0.5 + j0.5)\, I_1 - (2.5 + j4.33 + 0.5 + j0.5) = E_{b'a'}$$

$$-(2.5 + j4.33 + 0.5 + j0.5)\, I_1 + (0.5 + j0.5 + 2.5 + j4.33 + 5 + j0 + 0.5 + j0.5)\, I_2 = E$$

or, $6.08 \angle 9.46\, I_1 - 5.69 \angle 58.15\, I_2 = 100 \angle 120^\circ$

$$-5.69 \angle 58.15\, I_1 + 10.03 \angle 32.09\, I_1 = 100 \angle 120^\circ$$

$$\therefore \quad I_1 = \frac{\begin{vmatrix} 100 \angle 0 & -5.69 \angle 98.15 \\ 100 \angle 120 & 10.03 \angle 32.09 \end{vmatrix}}{\begin{vmatrix} 6.08 \angle 9.46 & -5.69 \angle 58.15 \\ -5.69 \angle 58.15 & 10.03 \angle 32.09 \end{vmatrix}}$$

$$= \frac{849.76 + j532.84 - 568.7 + j18.37}{45.64 + j40.45 + 14.34 - j29.02}$$

$$= 10.13 \angle 52.1$$

$$I_2 = \frac{\begin{vmatrix} 6.08 \angle 9.46 & 100 \angle 0 \\ -5.69 \angle 58.15 & 100 \angle 15 \end{vmatrix}}{61.06 \angle 10.8}$$

$$= \frac{386.41 + j469.42 + 300.26 + j433.33}{61.06 \angle 10.8}$$

$$= \frac{956.64 \angle 95.17}{61.06 \angle 10.8} = 15.67 \angle 84.37$$

9.18 Equivalent Y-loads of

loads as follows.

$$Z_{an} = \frac{Z_{ab} Z_{ac}}{Z_{ab} + Z_{bc} + Z_{ca}}$$

$$= \frac{50 \times 5 \angle -60^\circ}{5 \angle 0 + 5 \angle 60 + 5 \angle -60}$$

$$= 2.5 \angle 60^\circ$$

$$Z_{ab} = 5 \angle 0 = Z_{bc} = 5 \angle 60^\circ$$

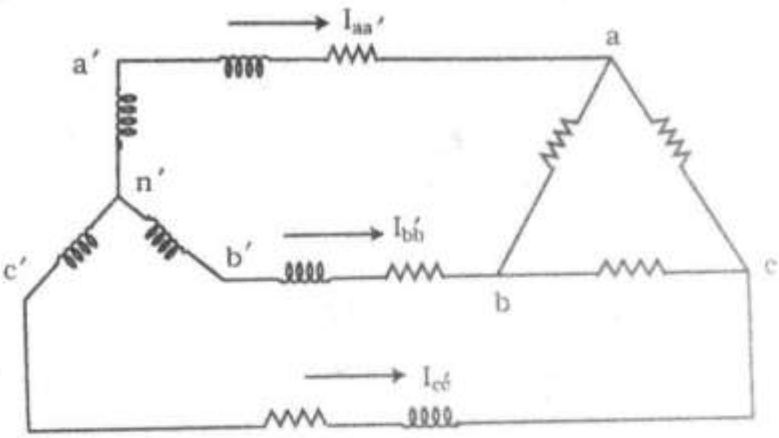
$$Z_{ca} = 5 \angle -60^\circ$$

$$Z_{ab} + Z_{bc} + Z_{ca} = 10 \angle 0,\ Z_{bn} = \frac{Z_{ab} \times Z_{bc}}{10 \angle 0} = \frac{5 \angle 0 \times 5 \angle r0}{10}$$

$$= 2.5 \angle 60°$$

$$Z_{cn} = \frac{Z_{ca} Z_{bc}}{10 \angle 0} = \frac{5 \angle -60 \times 5 \angle 60}{10} = 2.5 \angle 0$$

9.19.

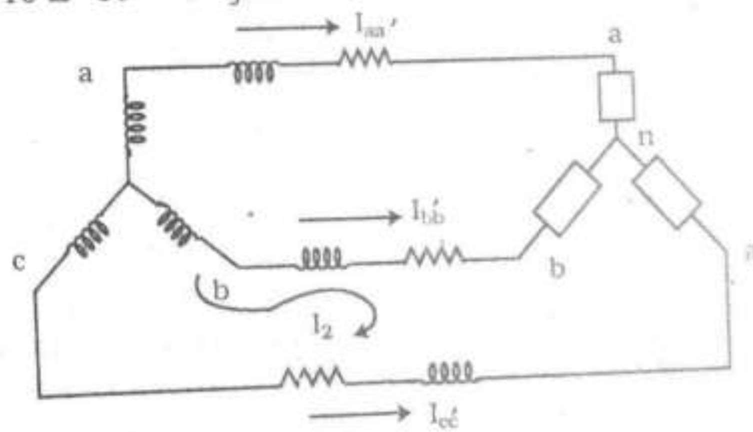


I_n fig. 1 :

$Z_{ab} = 10 \angle 0 = 10 + j0$

$Z_{bc} = 10 \angle 60 = 5\ j866$

$Z_{ca} = 10 \angle -60° = 5 - j866$



I_n fig. 2 :

$$Z_{an} = \frac{10 \angle 0 \times 10 \angle -60}{10 + 5\ j866 + 5 - j866} = 5 \angle -60 = 2.5 - j4.33$$

$$Z_{bn} = \frac{10 \angle 0 \times 10 \angle 60}{20} = 5 \angle 60 = 2.5 + j4.33$$

Now, $I_{a'a} = I_1 = 10.13 \angle 52.1$ amp

$$I_{b'b} = I_2 - I_1 = 15.67 \angle 84.37 - 10.13 \angle 52.1$$
$$= 1.54 + j15.59 - 6.22 - j7.99$$
$$= 8.93 \angle 121.62 \text{ amp}$$

Now, from fig. 1 : $V_{cb} + I_{a'a} - Z_{a'a} - I_{b'b}Z_{b'b} = E_{b'a'}$

or, $V_{ab} = 100 + j0 - (10.13 \angle 52.1)(0.5 + j0.5) + (8.93 \angle 121.62) \times (0.5 + j0.5)$

$$= 100 + j0 + 0.89 - j7.11 - 6.14 + j1.46$$
$$= 94.92 \angle -3.41^\circ$$

$$\therefore \quad I_{ab} = \frac{V_{ab}}{Z_{ab}} = \frac{94.92 \angle -3.41}{10 \angle 0} = 9.49 \angle -3.41 \text{ amp}$$

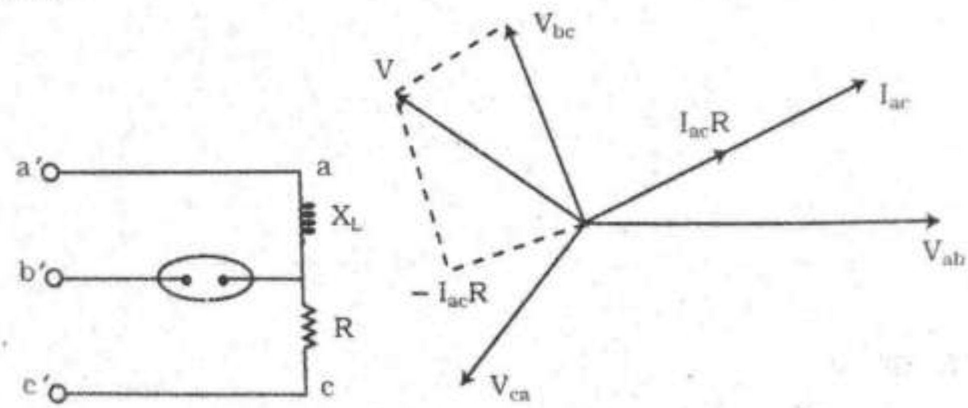
Now, $I_{ab} + I_{ac} = I_{a'a}$

or, $I_{ca} = I_{ab} - I_{a'a} = 9.47 - j56 - 6.22 - j7.99$

$$= 9.15 \angle -69.19^\circ \text{ amp}$$

$$I_{bc} + I_{ba} = I_{b'b} \text{ or, } I_{bc} = I_{b'b} + I_{ab}$$
$$= -4.68 + j7.6 + 9.47 - j0.56$$
$$= 8.52 \angle 55.77$$

9.20



The sequence is ab – ca – bc

Let, the magnitude of line voltage be 141.4 volts

$X_L = 100$ ohm

$R = 100$ ohm

$\therefore \quad V_{ab} = 141.4 \angle 0$

$V_{ca} = 141.4 \angle -120$

$V_{ca} = 141.4 \angle 120$

Since tne current through the voltmeter is negligible, we can assume X_L and R are connected in series across the voltage V_{ac}.

$$\therefore \quad I_{ac} = \frac{V_{ac}}{Z_{ac}} = \frac{-141.4 \angle -120^\circ}{100 + j100} = \frac{141.4 \angle 60}{141.4 \angle 45} = 1 \angle 15^\circ$$

Now, from KVL $- V_{bn} + V_{nc} = V_{bc}$ or, $V_m = V_{bc} - I_{ac}R$

$$= 141.4 \angle 120^\circ - 1 \angle 15 ; 100 = -70.7 + j122.46 - 96.59 - j25.98$$

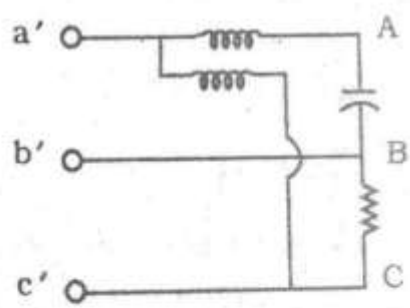
$$= 193.17 \angle 150 \text{ volts}$$

$\therefore$ The voltmeter read above the line voltage.

9.21 Not important.

9.22

From prob 13 $I_{A'A} = I_{AB} = 5 \angle 90^\circ$



$$V_{AC} = V_{AB} + V_{BC} = V_{AB} - V_{CB}$$

$$= 115 \angle 0 - 115 \angle -90^\circ$$

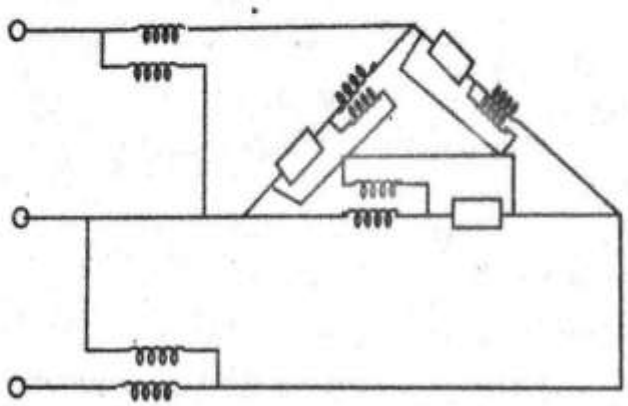
$$= 115 + j0 + 0 + j115$$

$$= 162.63 \angle 45^\circ$$

$$\therefore \quad W_{Ac-A'A} = V_{AC} I_{A'A} \cos\theta \Big]_{A'A}^{AC} = 162.63 \times 5 \times \cos(45 - 90)$$

$$= 575 \text{ watts}$$

9.23



Given,

$V_{ab} = 20$ V

$V_{bc} = 141.4$ V

$V_{ca} = 141.4$ V

Sequence

ab – bc – ca

$Z_{ab} = Z_{bc} = Z_c$

$= (8 - j6)$

Since the sequence is ab – bc – ca

$\therefore \quad V_{ab} = 200 \angle 0$

$V_{bc} = 141.4 \angle -120°$

$V_{ca} = 141.4 \angle 120$

$Z_{ab} = Z_{bc} = Z_{ca} = 8 - j6$

$= 10 \angle -36.87°$

$$I_{ab} = \frac{V_{ab}}{Z_{ab}} = \frac{200 \angle 0}{10 \angle -36.87} = 20 \angle 36.97°$$

$$I_{bc} = \frac{V_{bc}}{Z_{bc}} = \frac{141.4 \angle -120}{10 \angle -36.87} = 14.14 \angle -83.13$$

$$I_{ca} = \frac{V_{ca}}{Z_{ca}} = \frac{141.4 \angle 120}{10 \angle -36.87} = 14.14 \angle 15.87°$$

$W_{ab} = V_{ab} I_{ab} \cos\theta = 200 \times 20 \cos(0 - 36.87) = 3200$ watts

Now, $I_{a'a} = I_{ab} - I_{ca} = 18 + j12 - 13 - j5.55 = 29.7 \angle 12.54$

$I_{c'c} = I_{ca} - I_{bc} = -13 - 15.55 - 1.89 + j14 = 24.45 \angle 126.92°$

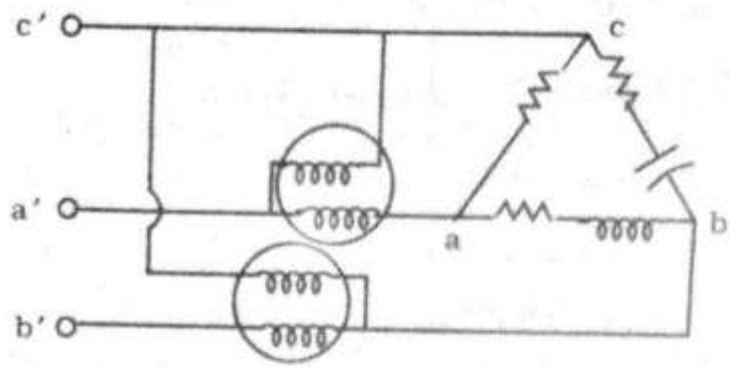
Now $W_a = V_{ab} I_{a'a} \cos\theta \Big]_{I_{a'a}}^{V_{ab}} = 200 \times 29.7 \cos(0 - 12.54)$

$= 5798.3$ watts

$W_b = V_{ab} I_{c'c} \cos \Big]_{I_{c'c}}^{V_{cc'}} = 141.4 \times 24.45 \cos(60 - 121.92)$

$= 1627.3317$ watts

9.24



Since the voltage sequence is ab – ca – bc, from example 1

We have

(a) $W_a = V_{ac} I_{a'a} \cos\theta \Big]_{I_{a'a}}^{V_{ac}}$ $V_{ac} = -V_{ca} = -100 \angle -120$

$= 100 \angle 60°$

So,

$W_a = V_{ac} I_{a'a} \cos\theta \Big]_{I_{a'a}}^{V_{ac}}$

$= 100 \times 9.26 \times \cos(60 - 23.4)$

$= 106.13$ watts

$W_b = V_{bc} I_{b'b} \cos\theta \Big]_{I_{b'b}}^{V_{bc}}$

$= 100 \times 2.9 \times \cos(120 - 146.9)$

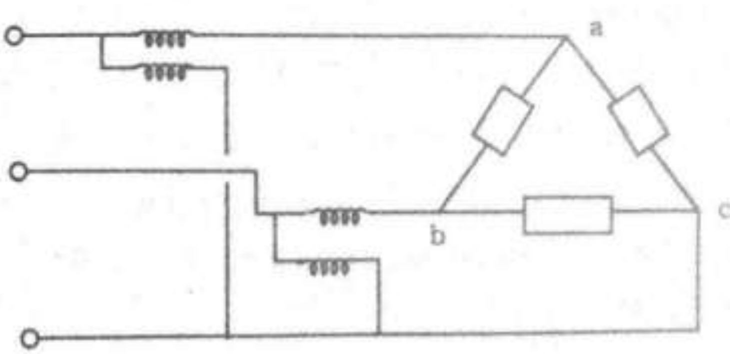
$= 2586.2$ watts

(b) If W_a and W_b are varmeters then their reading will be

$P_{xa} = 100 \times 9.26 \sin(60 + 23.4) = 919.86$ var (indctive)

$P_{xb} = 100 \times 29 \sin(120 - 145.9) = 1312.06$ vars (capacitive)

9.25



Taking calculations from problem 11 we have

(a) $W_a = V_{ac} I_{a'a} \cos\theta \left]\begin{matrix} V_{ac} \\ I_{a'a} \end{matrix}\right. = 100 \times 17.32 \cos(120 - 90^\circ) = 1500$ watts

$W_b = V_{bc} I_{b'b} \cos\theta \left]\begin{matrix} V_{bc} \\ I_{b'b} \end{matrix}\right.$

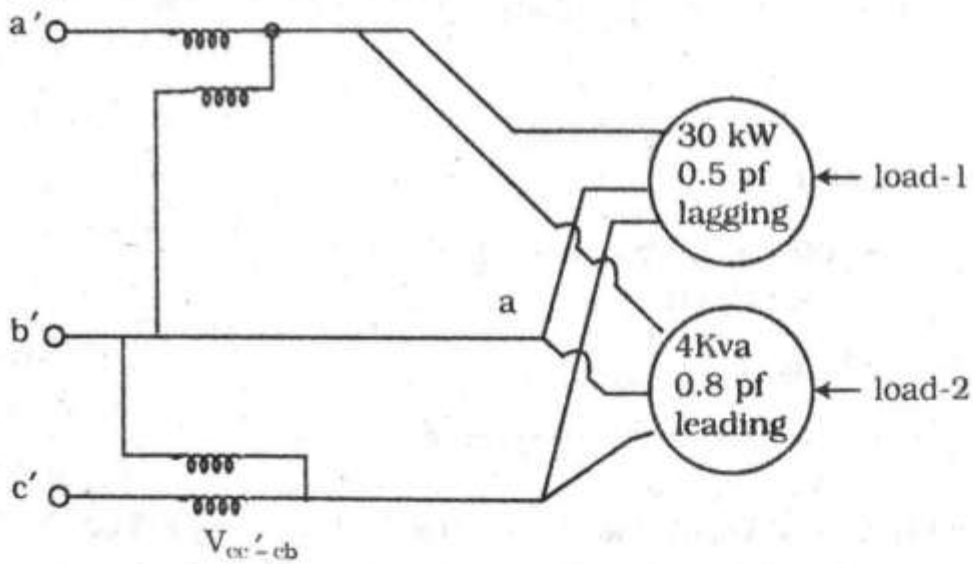
$= 100 \times 17.32 \times \cos(180 + 150^\circ)$

$= 1500$ watts

(b) $P_{xc} = 100 \times 17.325 \sin(120^\circ - 90^\circ) = 866$ vars (inductive)

$P_{xb} = 100 \times 17.32 \sin(180 + 150^\circ) = 866$ vars (capacitive)

9.26



Since the phase sequence is a′b′ – b′c′ – c′a′

$V_{ab} = V_{a'b'} = 200 \angle 0$, $V_{bc} = V_{b'c'} = 200 \angle -120^\circ$

$V_{ca} = V_{c'a'} = 200 \angle 120^\circ$

Let us assume that both the three phone loads are connected

$\therefore$ Per phase current in load 1 is, $I_{\varphi 1} = \dfrac{3000}{j \times 200 \times 0.5}$

$= 10$ amp

and these phase currents lag the applied voltage by $\cos^{-1} 0.5 = 60^\circ$

Per phase current in load 2 is $I_{\varphi 2} = \dfrac{4000}{3 \times 200} = 6.67$ amp

and these phase currents load the applied volt by

$\cos^{-1} 0.8 = 36.87°$

The unity p.f load current is in phase with V_{bc}

∴ Since these loads are parallel.

$$\therefore\ I_{ab} = 10 \angle(0 - 60°) + 6.67 \angle(0 + 36.87)$$

$$= 5 - j8.66 + 5.34 + j4$$

$$= 11.34 \angle -24.2°$$

$$I_{bc} = 10 \angle(-120 - 60) + 6.67 \angle(120 + 36.87) \ \frac{4000}{200} \times \angle -120$$

$$= -10 + j0 + 0.8 - j6.62 - 10 - j17.32 = 30.69 \angle -128.73$$

$$I_{ca} = 10 \angle(120 - 60°) + 6.67 \angle(120° + 36.87°)$$

$$= 5 + j8.66 - 6.13 + j2.68 = 11.34 \angle 95.72$$

$$\therefore\ I_{a'a} = I_{ab} - I_{ca} = 10.34 - j4.66 + 1.13 - j11.28$$

$$= 1964 \angle -54.26°$$

$$I_{c'c} = I_{ca} - I_{bc} = -1.23 + j11.28 + 19.2 + j23.94$$

$$= 39.59 \angle 62.84° = 2294.38 \text{ watts}$$

$$W_{c'c-cb} = 200 \times 39.59 \cos(60 - 62.84)$$

$$= 7908.28 \text{ watts}$$

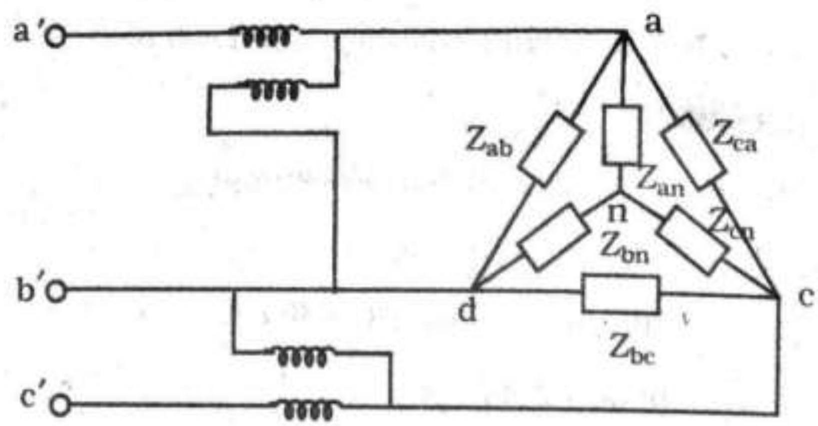
$$V_{cb} = -V_{bc} = -200 \angle -120° = 200 \angle 60°$$

(b) $\Sigma VI \cos\theta = 2294.38 + 7908.28 = 10202.66$ watts

$\Sigma VI \sin\theta = 3183.26 - 392.31 = 2795.95$ vars

$$\therefore\ \text{Vector p.}f = \frac{10202.66}{\sqrt{(10202.66)^2 + (2795.95)^2}} = 0.964$$

9.27



$Z_{an} = 10 \angle 0 \; Z_{bn} = 10 \angle -60°$

$Z_{cn} = 10 \angle 90°$

The voltage sequence is—

$a'b' - c'a' - b'c'$

$V_{ab} = V_{a'b'} = 100 \angle 0$

$V_{bc} = V_{b'c'} = 100 \angle 120$

$V_{ca} = V_{c'a'} = 100 \angle -120$

Transfering from Y

$\therefore \quad W_{a'a-ab} = 200 \times 14.14 \cos(0 + 45°) = 2000$ watts

$\therefore \quad W_{c'c-cb} = 200 \times 10 \cos(60 - 30) = 1732$ watts

9.29 Since the phase sequence is $n'a' - n'b' - n'c'$

Let

$E_{n'a'} = 115.4 \angle 0, \; E_{n'b'} = 115.4 \angle -120°, \; E_{n'c'} = 115.4 \angle 120$

$V_{ab} = E_{n'a'} - E_{n'b'} = 115.4 \angle 0 - 115.4 \angle -120° = 15.4 \angle 120°$

$V_{bc} = E_{n'b'} - E_{n'c'} = 115.4 \angle -120° - 115.4 \angle 120°$

$= 57.7 - j100 + 57.7 - j100$

$= 200 \angle -90°$

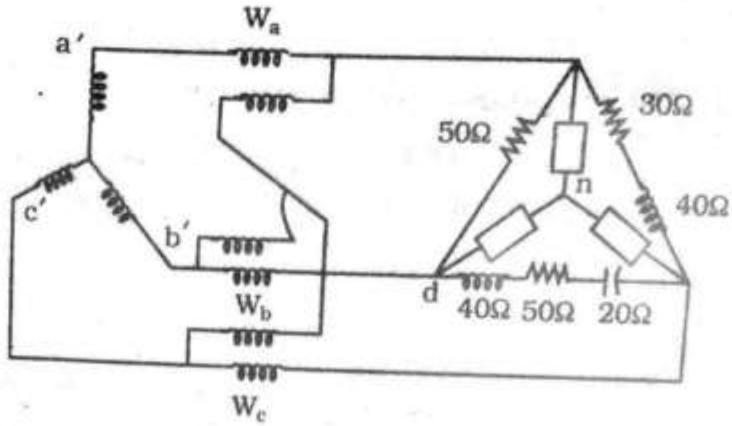
$V_{ca} = E_{n'c'} - E_{n'a'} = -57.7 + j100 - 115.4 = 200 \angle 150$

Taking V_{ab} as reference we can write—

(a) $V_{ab} = 200 \angle 0$ $\qquad E_{n'a'} = 175.4 \angle -30°$

$V_{bc} = 200 \angle(-90° - 30°) = 200 \angle -120°$ $\qquad E_{n'b'} = 115.4 \angle -150$

$V_{ca} = 220 \angle(-150 - 30°) = 200 \angle 120°$ $\qquad E_{n'c'} = 115.4 \angle 90°$



Z_{ab} = 50 ohm, $Z_{bc} = 40 + j(50 - 20) = 50 \angle 36.87°$

$Z_{ca} = 30 + j40 = 50 \angle 53.13°$

(b) $I_{ab} \because \frac{V_{ab}}{Z_{ab}} = \frac{200 \angle 0}{50 \angle 0} = 4 \angle 0$ amp

$$I_{bc} = \frac{V_{bc}}{Z_{bc}} = \frac{200 \angle -120}{50 \angle 36.87} = 4 \angle -156.87° \text{ amp}$$

$$I_{ca} = \frac{200 \angle 120°}{50 \angle 53.13} = 4 \angle 66.87° \text{ amp}$$

(c) $I_{a'a} = I_{ab} - I_{ca} = 4 - 1.57 - j3.68 = 4.41 \angle -56.56°$

$I_{b'b} = I_{bc} - I_{ab} = -3.68 - j1.57 - 4 = 7.84 \angle -168.4°$

$I_{c'c} = I_{ca} - I_{bc} = 1.57 + j3.68 + 3.62 + j1.57 = 7.42 \angle 45°$

(d) Transferring term Δ to Y we get,

$$Z_{an} = \frac{50 \times 50 \angle 53.13°}{50 + 40 + j30 + 30 + j40} = \frac{50 \times 50 \angle 53.13}{138.92 \angle 30.26} = 18 \angle 22.87°$$

$$Z_{bn} = \frac{50 \times 30 \angle 36.87}{138.92 \angle 30.26} = 18 \angle 6.61$$

$$Z_{cn} = \frac{50 \angle 36.87 \times 50 \angle 53.13}{138.92 \angle 30.26} = 18 \angle 59.74°$$

Now, $W_a + W_b + W_c = \Sigma V_{w\Delta} I_{w\Delta} \cos\theta$

$= 79.38 \times 4.41 \cos(-33.69 + 56.56) + 14.12 \times 7.84 \times \cos$

$(-161.79 + 168.4) + 133.56 \times 7.42 \cos(104.74 - 45)$

= 1920.97 watts

(e) $W_a = V_{ab} I_{a'a} \cos\theta = 200 \times 4.41 \cos(0 + 56.56) = 486.04$ watts

$W_b = V_{b'b} \cos\theta$ = Do yourself

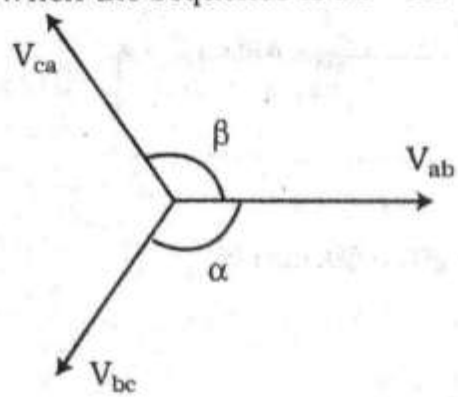
$W_c = V_{cb} I_{c'c} \cos\theta = 200 \times 7.42 \cos(60 - 45) = 1435$ watts

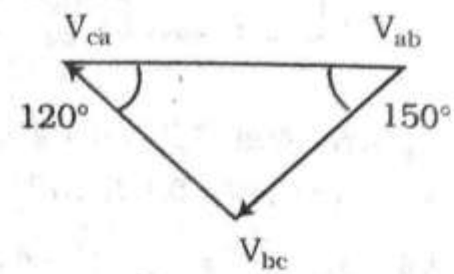
9.30

There may be two types of voltage sequence :

(i) ab – bc – ca (ii) ab – ca – bc

When the sequence is ab – bc – ca





$$\cos \alpha = \frac{200^2 + 150^2 - 120^2}{2 \times 200 \times 150}$$

$\therefore \quad \alpha = 36.71°$

Since α is measured in clockwise direction.

$$= -(180 - 36.71) = -143.29°$$

$$\cos \beta = \frac{200^2 + 120^2 - 150^2}{2 \times 200 \times 120}, \ \beta = 48.35°$$

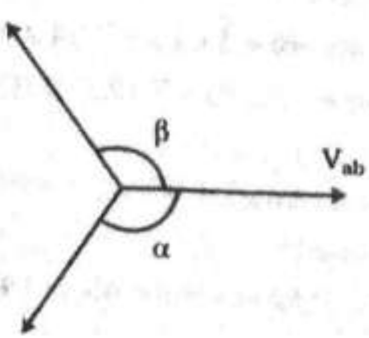
Since β is measured in anticlockwise direction,

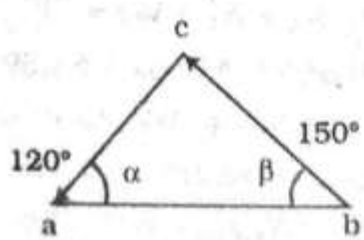
$\therefore \quad \beta = 180 - 48.35 = 131.65°$

$\therefore V_{ab} = 200 \angle 0, V_{bc} = 150 = 150 \angle -143.29°,$

$V_{ca} = 120 \angle 131.65°$

(ii) When the sequence is ab – ca – bc





Here, $\cos \alpha = \dfrac{200^2 + 120^2 - 150^2}{2 \times 200 \times 120}$

$\therefore \quad \alpha = 48.35°$

Since α is measured in the clockwise direction.

$\therefore \quad \alpha = (180 - 48.35) = -131.65°$, $\cos \beta = \dfrac{220^2 + 150^2 - 120^2}{2 \times 200 \times 150}$

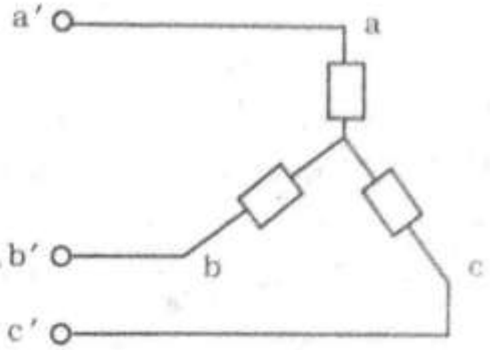
$\therefore \quad \beta = 36.71°$

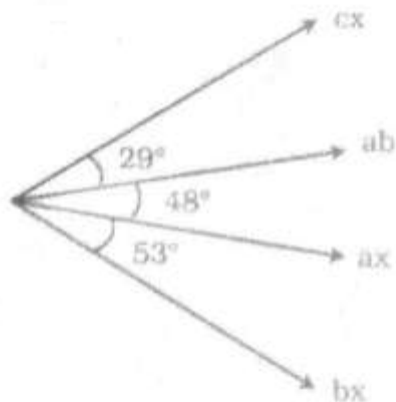
Since is measured in the anticlockwise direction

$\beta = 180 - 36.71 = 143.29°$

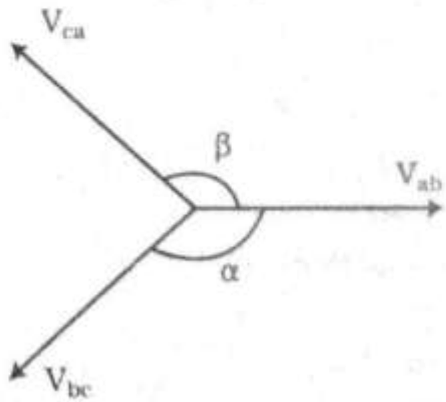
$\therefore \quad V_{ab} = 200 \angle 0,\ V_{ca} = 120 \angle -131.65,\ V_{bc} = 150 \angle 143.29°$

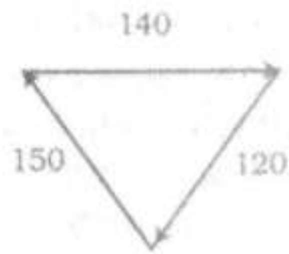
9.31





Since the voltage sequence is abc





$\cos \alpha' = \dfrac{140^2 + 120^2 - 150^2}{2 \times 140 \times 120} \therefore \alpha = 70°$

Since α is measured in the clockwise direction

$\therefore \quad \alpha = -(180 - 70) = -110^\circ$

$$\cos\beta = \frac{140^2 + 150^2 - 120^2}{2 \times 140 \times 150}$$

$\therefore \quad \beta = 48.74^\circ$

$\therefore \quad \beta = 180 - 48.74 = 131.26^\circ$

$\therefore \quad V_{ab} = 140 \angle 0^\circ$

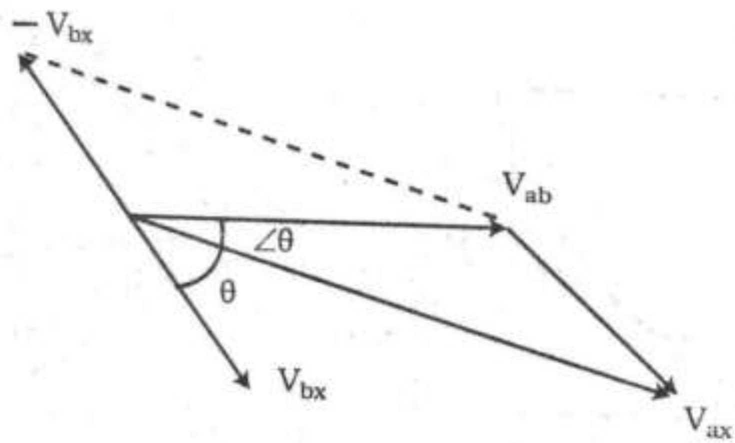
$V_{bc} = 120 \angle -110^\circ$

$V_{ca} = 150 \angle 131.26^\circ$

Let,

$V_{an} = 200 \angle 0,\ V_{bn} = 80 \angle \varphi$

$V_{cn} = 104.2 \angle r$



Now, $V_{an} - V_{bn} = V_{ab}$

or, $200 \angle 0 - 80 \angle \varphi = 140 \angle 0$

$\therefore \quad 200 \cos\theta - 80 \cos\varphi = 140$ .... (i)

$200 \sin\theta - 80 \sin\theta = 0$ .... (ii)

[Equiting real and imaginary parts]

or, $10 \sin\theta = 4 \sin\varphi$ [From equation (ii)]

or, $10^2 \sin^2\theta = 16(1 - \cos^2\theta)$

or, $10^2(1 - \cos^2\theta) = 16(1 - \cos^2\theta)$

or, $10^2 - 10^2\cos^2\theta = 16 - 16\cos^2\varphi$

or, $16\cos^2\varphi = 16 - 10^2 + 10^2\cos^2\theta = 10^2\cos^2\theta - 84$

From (i) $\Rightarrow 10 \cos \theta + 7 = 4 \cos \varphi$

or, $4 \cos^2 \varphi = (10 \cos \theta + 7)^2$

So, $10^2 \cos^2 \theta - 84 = 100 \cos^2 \theta + 2 \times 10 \times 7 \cos \theta + 49$

or, $10^2 \cos^2 \theta = 100 \cos^2 \theta + 140 \cos \theta + 49 + 84$

or, $\cos \theta = \frac{133}{140} = 0.95$

$\therefore \quad \theta = 18.1948$

From (ii) $200 \sin(-18.2) = 80 \sin \varphi$

$\varphi = -51.34°$

Again,

$V_{bn} - V_{cn} = V_{bc}$

$\therefore \quad 80 \sin \varphi - 104.2 \sin \sigma = -112.76$

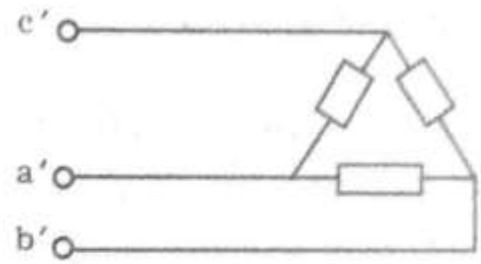
or, $\sigma = 28.9°$

$\therefore \quad V_{an} = 200 \angle -18.2°$

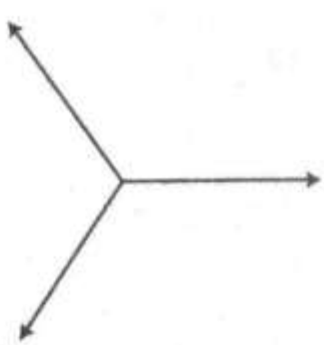
$V_{bn} = 80 \angle -51.34°$

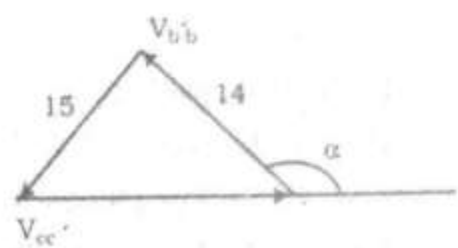
$V_{cn} = 104.2 \angle 28.9°$

9.32



$I_{a'c} + I_{b'b} + I_{c'c} = 0$





$I_{a'a} = 20$, $I_{b'b} = 14$, $I_{c'c} = 15$

Let us first take $I_{a'a}$ as reference

$\therefore \quad \cos\alpha' = \dfrac{20^2 + 14^2 - 15^2}{2 \times 20 \times 4}$

$\therefore \quad \alpha = 48.5°$

$= 180° - 48.5$

$= 131.5°$

$\cos\beta' = \dfrac{20^2 + 15^2 - 14^2}{2 \times 20 \times 15}$

$\therefore \quad \beta' = 44.36°$

$\therefore \quad \beta = -180° + 44.36 = -135.64$

$\therefore \quad I_{a'a} = 20\angle 0$, $I_{b'b} = 14\angle 131.5°$, $I_{c'c} = 15\angle -135.64°$

Let, $I_{ab} = 12\angle 0$, $I_{bc} = 2\angle\varphi$, $I_{ca} = 15\angle r$ $I_{ab} - I_{ca} = I_{a'a}$

or, $12\angle 0 - 15\angle r = 20\angle 0$

or, $12\cos\theta - 15\cos r = 20$ .... (i)

$12\sin\theta - 15\sin r = 0$ .... (ii)

$\Rightarrow \quad 15^2\cos^2 r = 15^2 - 12^2 + 12^2\cos^2\theta$

From (i)

$\Rightarrow \quad 12^2\cos^2\theta - 24 \times 20\cos\theta + 20^2 = 15^2 - 12^2 + 12^2\cos\theta$

$\therefore \quad \theta = -48.35°$

From (ii)

$\Rightarrow \quad 12\sin(-48.35) = 15\sin r$

$\therefore \quad r = -143.29$

Again,

$I_{bc} - I_{ab} = I_{b'b}$ or, $2\angle\varphi - 12\angle\theta = 14\angle 131.5$

$\therefore \quad 2\cos\theta - 12\cos\theta = -9.277$

$\therefore \quad 2\cos\theta = -9.277 + 12\cos(48.35)$

$\therefore \quad \varphi = 130.62 \therefore I_{ab} = 12\angle -48.35°$

$\therefore \quad I_{bc} = 2\angle 130:62$

and $I_{ca} = 15\angle -143.29°$

Now transfering the reference from $I_{a'a}$ to I_{ab} by adding 48.35° we have—

$I_{a'a} = 20 \angle 48.35°$, $I_{b'b} = 14 \angle 179.85°$

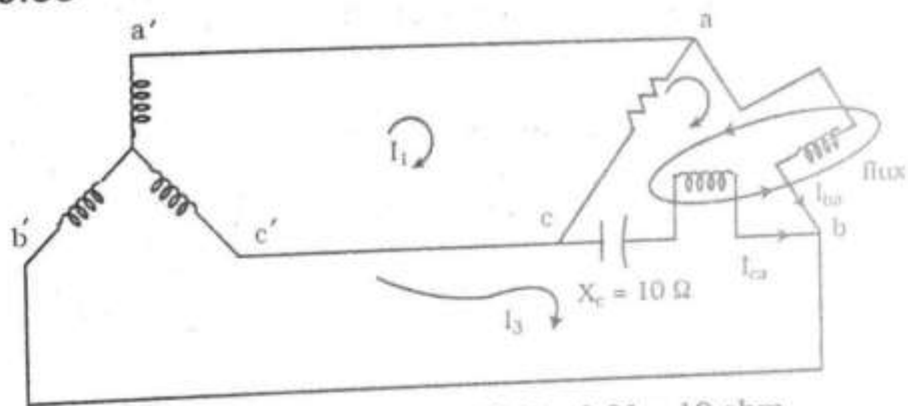
$I_{c'c} = 15 \angle -87.29$, $I_{ab} = 12 \angle 0$

$I_{bc} = 2 \angle 179°$, $I_{ca} = 15 \angle -94.95°$

9.33 Same as problem 16.

9.34 More or less same as example 13.

9.35



$L_{ab} = L_{cb} = 0.01$ H. $X_{Lab} = X_{Lcb} = 1000 \times 0.01 = 10$ ohm

$X_M = 0.5\sqrt{10 \times 10} = 5$ ohm

The loop equations are

$$10\, I_1 - 10\, I_2 = 57.7 \angle 90 - 57.7 \angle 210°$$

or, $I_1 - I_2 = 10 \angle 60°$ .... (i)

$$-10\, I_1 + [10 + j(10 + 10 + 5 + 5)]\, I_2 - j(10 - 10 + 5)\, I_3 = 0$$

or, $-10\, I_1 + 22.36 \angle 63.43\, I_2 + 5 \angle -90\, I_3 = 0$ ...... (ii)

$$-j(10 - 10 + 5)\, I_2 + (j10 - j10)\, I_3$$

$$= 57.7 \angle 210 - 57.7 \angle -30°$$

or, $5 \angle -90°\, I_2 = 100 \angle 180°$ .... (iii)

$\therefore\quad I_2 = 20 \angle 270°$

$= 20 \angle -90°$

From (i) $I_1 = 10 \angle 60° + 20 \angle -90° = 5 + j8.66 + 0 - j20$

$= 12.39 \angle -66.21$

From (ii) $5 \angle -90° \, I_3$

$= 10 \times 12.39 \angle -66.21 - (22.36 \angle 63.43) \times (20 \angle -90°)$

$= 50 - j113.37 - 400 + j200 = 360.56 \angle 166.1$

$\therefore \quad I_3 = 72.11 \angle 256.1°$

The line currents are

$I_{a'a} = I_1 = 12.39 \angle -66.21°, \; I_{b'b} = -I_3$

$= -72.11 \angle 256.1 = -72.11 \angle 76.1$

$I_{c'c} = I_3 - I_1 = 17.32 - j70 - 5 + j11.34 = 62.76 \angle -110.83°$

The phase currents are $-\; I_{ac} = I_1 - I_2$

$= 5 - j11.34 + j20 = 10 \angle 60°$

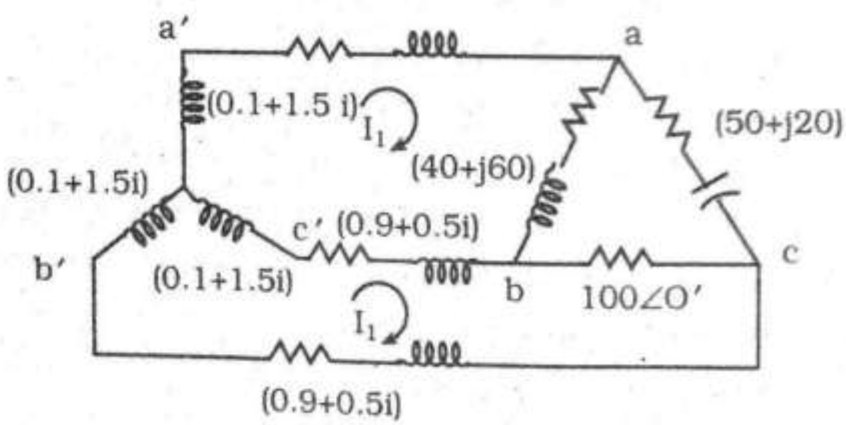
$I_{cb} = I_3 - I_2 = 72.11 \angle 256.1 + j20 = -17.32 - j70 + j20$

$= 52.91 \angle -109.1$

$\therefore \quad I_{ba} = -I_2 = -20 \angle -90° = 20 \angle 90°$

36. Similar to 35.

37.



$Z_{ab} = 40 + j60$

$Z_{bc} = 100$

$Z_{ca} = 50 - j20$

$Z_{a'n} = 0.1 + 1.5\,j = Z_{b'n} = Z_{c'n}$

Since only two + loop current is necessary,

$$-E_{an} + E_{bn} + (0.9 + 0.5j + 0.9 + 0.5j)\, I_1 + (-0.9 - 0.5j)$$

$$I_2 - 0.1 - 1.5j + 40 + j60 + 2 \times (0.1 + 1.5j) = 0$$

$$(2 + 44.6j)\, I_1 + (-1 - 2j)\, I_2 = E_{an} - E_b = 1350 \angle 0 - 675 - j1170$$

$$= 675 - 1170j$$

$$(2 + 44.6j)\, I_1 + (-1 - 2j)\, I_2$$

and, $+ E_{cn} - E_{bn} + (0.9 + 0.5j + 0.9 + 0.5j + 100 + 2 \times$

$$(0.1 + 1.5j)\, I_2 + (-0.9 - 0.5j - 0.1 - 1.5j)\, I_1 = 0$$

$$(102 + 4j)\, I_2 + (-1 - 2j)\, I_1 = E_{bn} - E_{cn}$$

$$= -675 - 1170j - (-675 + 1170j)$$

$$= -675 - 1170j + 675 - 1170j$$

$$= -2 \times 1170j = -2340j$$

So $(2 + 44.6j)\, I_1 + (-1 - 2j)\, I_2 = 675 - 1170j$

$$(102 + 4j)\, I_2 + (-1 - 2j)\, I_1 = -2340j$$

Solve this equation hence find out I_1 and I_2.

[Tips : In complex mode normal scientific calculator does not operate in equation solving but how can we do that ? Simple click CALC button after making equation for 2 × 2 Matric like (AB – CM) parameters in

Chapter–11

Electric Wave Filters

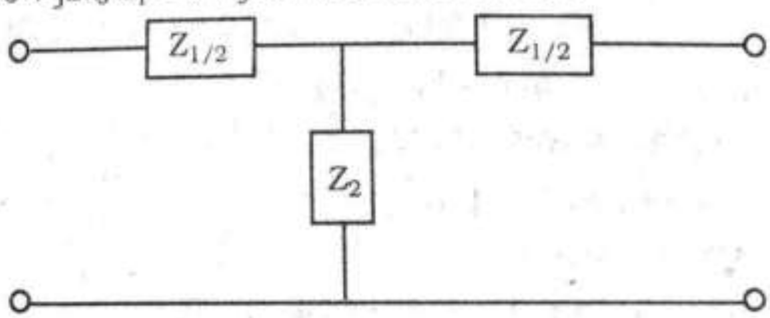
11.1 Here,

$L_1 = 0.047$ H : $Z_{OT} = ?$

$R_1 = 0$: at 50 cycle

$C_2 = 300\ \mu f$: at 100 cycle

$Z_{1/2} = 0 + j2\pi f L_1 = 0 + j2\pi \times 50 \times 0.0477 \angle 90^\circ$



$\therefore \quad Z_1 = 29.54 \angle 90^\circ$

$$Z_2 = \frac{1}{jwc_2} = \frac{10^6}{j2\pi \times 50 \times 300} = 10.61 \angle -90^\circ$$

$Z_1 Z_2 = 29.54 \angle 90^\circ \times 10.61 \angle -90^\circ = 313.42 \angle 0^\circ = 313.42 + j0$

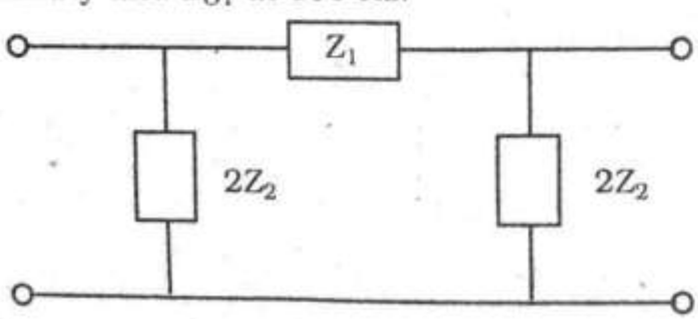
$$Z_1^2/4 = \frac{(29.54 \angle 90^\circ)^2}{4} = 218.153 \angle 180^\circ = -218.153$$

$Z_{OT} = \sqrt{Z_1 Z_2 + Z_1^2/4} = \sqrt{313.42 + j0 - 218.153 + j0}$

$= 95.267 \angle 0^\circ = 9.76 \angle 0^\circ$

$\therefore \quad Z_{oT} = 9.76 \angle 0$ ohm

11.2 Similarly find Z_{OT} at 100 Hz.



at 200 Hz

$Z_1 = jWL_1 = j2\pi \times 200 \times 0.02 = 25.13 \angle 90^\circ$

$$2Z_2 = \frac{1}{jwc_2/2} = -j\frac{1}{2\pi f c_{2/2}} = \frac{j \times 10^6}{2 \times 200 \times 2} = 397.89 \angle -90^\circ$$

$\therefore \quad Z_2 = 198.94 \angle -90^\circ$

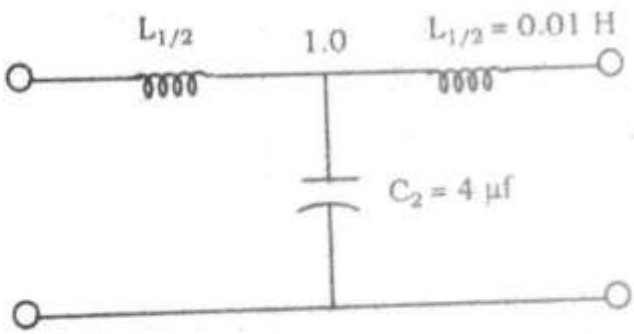
Now, $1 + \frac{Z_1}{4Z_2} = 1 + \frac{25.13 \angle 90^\circ}{4 \times 198.94 \angle 90^\circ} = 1 + 0.0315 \angle 180^\circ$

$= 0.9685 \angle 0^\circ$

$\therefore \quad Z_1Z_2 = 4999.36 \angle 0^\circ$

$$Z_{0\pi} = \sqrt{\frac{Z_1Z_2}{1 + Z_1/4Z_2}} = \sqrt{\frac{4999.36 \angle 0^\circ}{0.9685 \angle 0^\circ}} = 71.85 \angle 0^\circ$$

11.3



$Z_{1/2} = jWL_{1/2} = j \times 2\pi \times 200 \times 0.01 = 12.57 \angle 90^\circ$ ohm

$\therefore \quad Z_1 = 25.14 \angle 90^\circ$ ohm

$\therefore \quad Z_2 = \frac{1}{jwc_2} = \frac{10^6}{j \times 2\pi \times 200 \times 4} = 108.94 \angle -90^\circ$ ohm

$Z_{oc} = \frac{Z_1}{2} + Z_2 = 12.57 \angle 90^\circ + 198.94 \angle -90^\circ$

$Z_{oc} = 186.37 \angle -90^\circ$

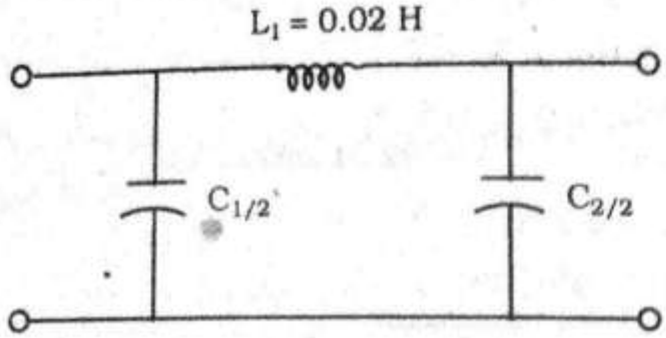
$$Z_{sc} = \frac{Z_{1/2} \times Z_2}{ZV_2 + Z_2} + Z_{1/2} = \frac{j12.57 \times -j98.96}{j12.57 - j198.94} + j12.57 \angle 90^\circ$$

$$= \frac{2500.6758}{186.37 \angle -90^\circ} + j12.57 = 25.99 \angle 90^\circ$$

$[\therefore Z_{sc} = 26 \angle +90^\circ$ ohm

$Z_\pi = \sqrt{Z_{oc}Z_{sc}} = 69.61 \angle 0^\circ$ ohm

11.4



$$Z_1 = jWL_1 = j \times 2\pi \times 200 \times 0.02 = 25.13 \angle 90°$$

$$\therefore \quad Z_1 = 25.13 \angle 90° \text{ ohm}$$

$$2Z_2 = \frac{1}{jwc_{2/2}} = \frac{j \times 10^6}{2\pi \times 200 \times 2.0} = 397.89 \angle -90°$$

$$\therefore \quad Z_2 = 198.94 \angle -90° \text{ ohm}$$

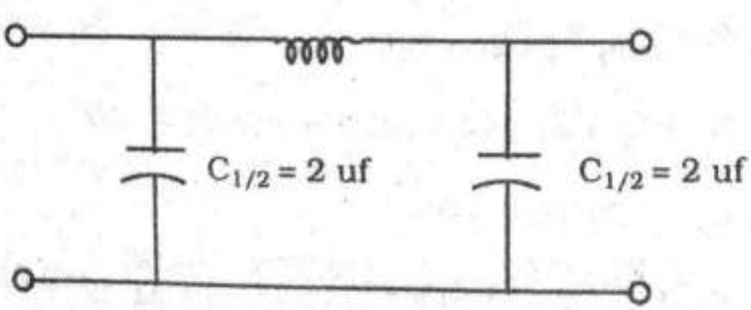
$$Z_{ac} = \frac{Z_1 \times 2Z_2}{Z_1 + 2Z_2} = \frac{(j25.13) \times (-j397.89)}{j25.13 - j397.83}$$

$$\therefore \quad Z_{sc} = \frac{(2Z_2 + Z_1) \times 2Z_2}{Z_1 + 4Z_2}$$

$$= \frac{-j372.76 \times -j397.89}{j25.13 - j795.76} = 196.46 \angle -90°$$

$$Z_{oc} = 192.49 \angle -90°$$

11.5 $Z_{o\pi} = \sqrt{Z_{oc}Z_{sc}} = 71.85 \angle 0° \Omega$



at 200 Hz

$$Z_1 = jWL_1 = 2\pi \times 200 \times 0.02j = j25.13, \; 2Z_2 = \frac{-j}{2\pi f c_{2/2}}$$

$$\text{or,} = \frac{-j \times 10^6}{2\pi \times 200 \times 2} = -j397.09$$

Now, $1 + Z_1/4Z_2 = 1 + \frac{25.13 \angle 0}{2 \times 307.89 \angle -90^\circ}$

$= 1 - 0.04\,j = 1 \angle 2.336$

$Z_1 Z_2 = (25.13 \angle 90)(-j397.89) = 9998.9757 \angle 0^\circ$

$Z_{0\pi} = \sqrt{Z_1 Z_2} = 99.994$ ohm

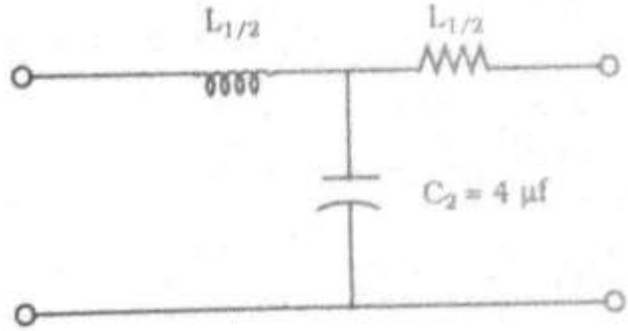
Now, $\frac{I_1}{I_2} = \frac{2Z_2 + Z_{0\pi}}{2Z_2 - Z_{0\pi}}$

$\frac{I_1}{I_2} = \frac{-j397.89 + 99.994}{-j397.89 - 99.994} = 1 \angle 28.213$

$\therefore \quad I_1/I_2 = 1 \angle 28.213$

For 2000 Hz, Similarly proceed

11.6



At 200 Hz

$\frac{Z_1}{2} = 12.57 \angle 90^\circ \pi$

$Z_2 = 198.94 \angle -90^\circ \pi$

$Z_{OT} = 69.61 \angle 0^\circ$ ohm

$\frac{I_1}{I_2} = \frac{Z_{oT} + Z_{1/2}}{Z_{oT} - Z_{1/2}} = \frac{69.61 + j12.57}{69.61 - j12.57} = \frac{70.74 \angle 10.24}{70.74 \angle -10.24} = 1.0 \angle 20.48^\circ$

$\therefore \quad I_1/I_2 = 1.11 \angle 113^\circ \Rightarrow I_1/I_2 = e^{\alpha + j\beta}$

$\therefore \quad e^{(\alpha + j\beta)} = 1.11 \angle 113$

or, $\quad e^{\alpha}.e^{j\beta} = 1.11 \angle 113$

or, $\quad e^{\alpha} = 1.11$

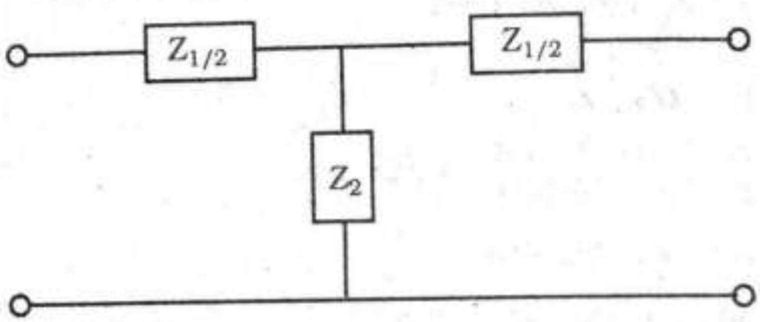
$\alpha = 0.1043$ neper

and $e^{j\beta} = 0.6277$ [in radian]

$j\beta = (0.6277)$ logarith of e bare

Do yourself.

11.9



$$\frac{Z_1}{2} = -j\ 10^6/wc_2 = \frac{-j \times 10^6}{1257 \times 7.96}$$

$Z_2 = 4 + j\omega L_1 = 4 + j1257 \times 0.0159 = 20.38 \angle 76.6\ \Omega$

at 200 Hz, $Z_1 = 200 \angle -90°$, $Z_2 = 20.4 \angle 78.7°$

$$\frac{Z_1}{4Z_2} = \frac{200 \angle -90°}{4 \times 20.4 \angle 78.7} = 2.45 \angle -168.7$$

$$\sqrt{\frac{Z_1}{4Z_2}} = 1.57 \angle -84.35 = 0.15 - j1.56$$

$$1 + \frac{Z_1}{4Z_2} = 1 + 2.45 \angle -168.7° = 1 - 2.40 - j0.48$$

$= -1.4 - j0.48$

$= 1.48 \angle -161$

$$\sqrt{1 + \frac{Z_1}{4Z_2}} = 1.22 \angle -80°.54° = 0.2 - j1.2$$

$$\therefore \quad \sqrt{1 + \frac{Z_1}{4Z_2}} + \sqrt{\frac{Z_1}{4Z_2}} = 0.2 - j1.2 + 0.15 - j1.56$$

$= 2.78 \angle -82.77°$

$\therefore \quad \alpha = 2.05$ neper $= 2.05 \times 8.68 = 17.8063$ db

and $\beta = 2.89$ rad/sec

11.10. Here $Z_1 = 200 \angle 90°$, $Z_2 = 50 \angle -90°$

$$\sqrt{\frac{Z_1}{4Z_2}} = \sqrt{\frac{200 \times 1 \angle 90°}{200 \angle -90°}} = 1 \angle 90°$$

$$\sqrt{1 + \frac{Z_1}{4Z_2}} = \sqrt{1 + 1 \angle 180°} = \sqrt{1 - 1} = 0$$

Now, $\alpha + j\beta = 2 \log (1 \angle 80°) = 2 \log e(1) + 2j \times \frac{90}{57.3}$

$\therefore \quad \alpha + j\beta = 0 + j3.141$

Hence $\alpha = 0$

$\beta = 3.14 = \pi$

11.11 $Z_1 = j\omega L_1$

$$Z_2 = \frac{1}{j\omega C_2}$$

$$\therefore \quad \frac{Z_1}{4Z2} = \frac{\omega^2 L_1 C_1}{4} \angle 180° \text{ if } \omega = 1.5/\sqrt{L_1 C_2}$$

$$\therefore \quad \frac{Z_1}{4Z_2} = \frac{-L_1 C_2}{4} \times \frac{1.5^2}{L_1 C_2} \times \frac{9^2}{L_1 C_2}$$

$$= 2.25 \angle 180° \text{ ohm} = -2.25 \text{ ohm}$$

(a) $$\sqrt{\frac{Z_1}{4Z_2}} = \sqrt{0.5625 \angle 180°} = 0.75 \angle 90°$$

$$\therefore \quad \sqrt{1 + \frac{Z_1}{4Z_2}} = \sqrt{1 - 0.5625 + j0}$$

$$= \sqrt{0.4375 \angle 0°}$$

$$= 0.6614 \angle 0°$$

$$\therefore \quad \alpha + j\beta = 2 \log [0.6614 + j0 + j0.75] = 2 \log [1 \angle 48.59]$$

$$= 2 \log 1 + j\frac{2 \times 48.59}{57.3} = 0 + j1.7$$

$\therefore \quad \alpha = 0, \beta = 1.7$ rad

11.12 $\frac{W_1}{W_2} = \frac{I_1^2}{I_2^2} = (e^{\alpha}.e^{j\beta})^2 = e^{2\alpha}.e^{j2\beta} = 1$

$\therefore \quad \alpha = 0$

Similary proced in the same way.

11.13 Given, R_k = 600 k ohm, f_c = 940 Hz

$R_K^2 = \frac{I_1}{c_2}$

$\therefore c_2 = \frac{L_1}{R_K^2}$

$F_c = \frac{1}{\pi\sqrt{L_1C_2}}$

$= \frac{1}{\pi\sqrt{\frac{L_1}{R_K^2} \times L_1}}$

$= \frac{R_K}{\pi L_1}$

$\therefore L_1 = \frac{R_K}{\pi f_c} = \frac{600}{\pi \times 940}$

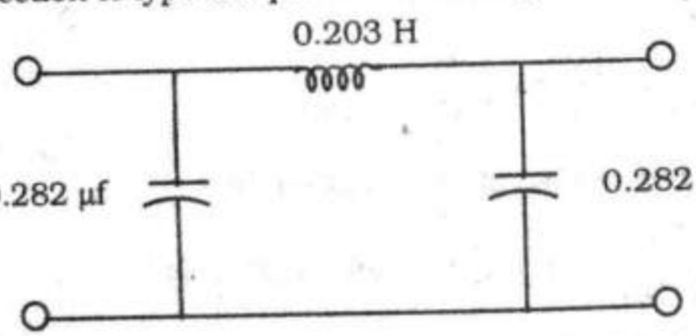
$= 0.203$ H

$\therefore C_2 = \frac{L_1}{R_K^2} = \frac{1}{\pi R_K f_C}$

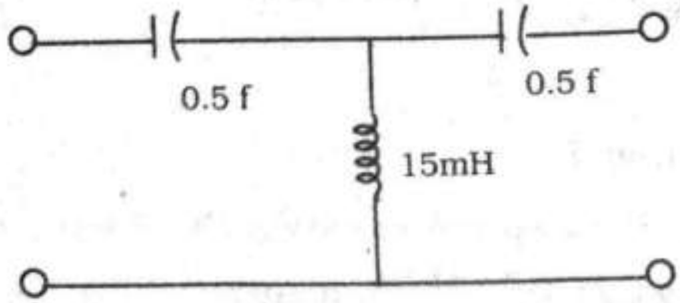
$= \frac{1}{\pi \times 600 \times 940}$

$= 0.564\ \pi f$

$\therefore$ T section K type law pass filter is diagnam.



11.16



$2C_{1K} = 1\mu f : L_{2K} = 15$ mH

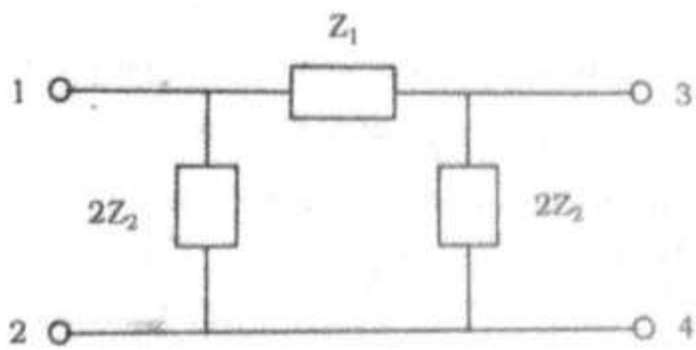
$\therefore C_{1K} = 0.5\mu f : = 0.015$ H

$$\therefore R_K = \sqrt{\frac{L_2 K}{C_{1K}}} = \sqrt{\frac{0.015}{0.5 \times 10^{-6}}} = 173.2 \text{ ohm}$$

$$\therefore \quad f_c = \frac{1}{4\pi\sqrt{L_2 C_{1K}}}$$

$$= \frac{1}{4\pi\sqrt{0.015 \times 0.5 \times 10^{-6}}}$$

$$= 919 \text{ Hz}$$



at 500 Hz

$$Z_1 = 50 + j2\pi \times 500 \times 100 \times 10^{-3} = 318.11 \angle 80.96^\circ$$

$$2Z_2 = -j\frac{10^6}{2\pi \times 500 \times 0.3} = 1061 \angle 80.96^\circ$$

$$\therefore \quad Z_{oc} = \frac{(Z_1 + 2Z_2)\, 2Z_2}{Z_1 + 2Z_2}$$

$$= \frac{(50 + j314.16 - j1061)\,(1061 \angle -90^\circ)}{50 + j314 - j212}$$

$$= \frac{794171.07 \angle -176.17}{1808.53 \angle -88.42}$$

$$= 439.12 \angle -87.75$$

$$Z_{sc} = \frac{Z_1 . 2Z_2}{Z_1 + 2Z_2}$$

$$= \frac{(318.11 \angle 80.96)\,(1061 \angle -90^\circ)}{50 + j314.16 - j1061}$$

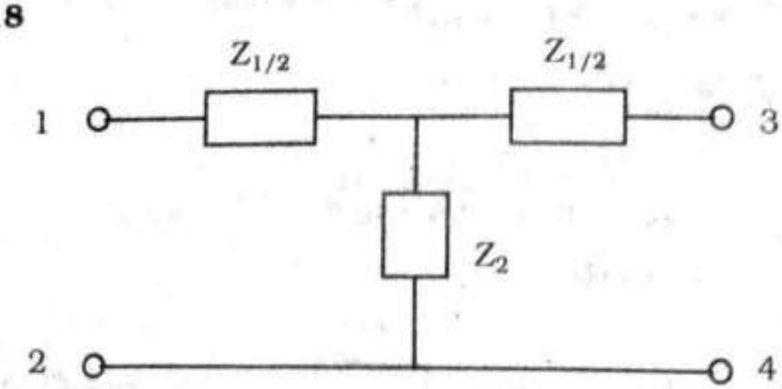
$$= \frac{337515.71 \angle -9.04}{748.51 \angle -86.17^\circ}$$

(b) Characteristics impedence $Z_{on} = \sqrt{Z_{sc} Z_{oc}}$

$$= 444.97 \angle -5.31$$

Similar calculation for other cycles.

11.18



(a) $\frac{Z_1}{2} = -j\frac{10^6}{2\pi \times 200 \times 0.6} = 2652.53 \angle -90°$

$Z_2 = jWL + R = 60 + j251.33 = 258.4 \angle 76.57$

$\therefore \quad \frac{Z_1}{4Z_2} = \frac{2652.58 \angle -90°}{4 \times 258.4 \angle 76.57} = 2.566 \angle -166.57$

Characteristics impedence $Z_{OT} = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}$

$= 1050.55 \angle -85.35°$

Propagation constant is r

$= \alpha + j\beta = 2 \log_e \left\{\sqrt{1 + \frac{Z_1}{4Z_2}} + \sqrt{\frac{Z_1}{4Z_2}}\right\}$

$= 2 \log_e \{\sqrt{1 - 2.496 - j0.596} + \sqrt{2.566 \angle -166.57}\}$

$= 2 \log_e \{1.269 \angle -79.14 + 1.6 \angle -33.235\}$

$= 2 \log_e \{0.239 - j1.25 + 0.187 - j1.589\}$

$= 2 \log_e \{2.871 \angle -81.47°\}$

$\therefore \alpha = 2.11$ neper and $\beta = -162.94°$

11.19 When the output terminals open circuited—

$R = 10$ ohm, $L = -190 \times 10^{-3}$ H, Z_{oc}

$= 10 + j2\pi \times 400 \angle -190 \times b^3)$

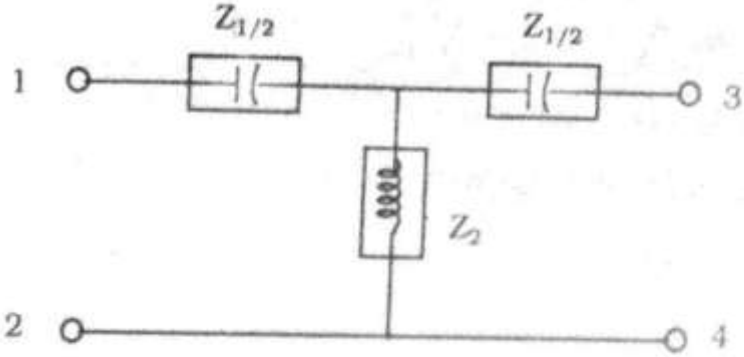
$= 10 - j477.52 = 477.63 \angle -88.8°$

Similarly, $Z_{SC} = 20 + j2\pi \times 400 \times (250 \times 10^{-3})$

$= 628.64 \angle 88.2$

$\therefore Z_{OT} = \sqrt{Z_{OC} Z_{SC}} = 547.96 \angle -0.3°$

11.20



(a) $\frac{Z_1}{2} = -j100$

$\therefore Z_1 = 200 \angle -90^\circ$

$Z_2 = j100 = 100 \angle 90^\circ$

$$Z_{OT} = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)} = \sqrt{Z_1 Z_2 + \frac{Z_1^2}{4}}$$

$$= \sqrt{(200 \angle -90^\circ)(100 \angle 90^\circ) + \left(\frac{200 \angle 90}{4}\right)^2}$$

$$= \sqrt{2000 - 1000} = 100 \angle 0$$

(b) At half frequency $- Z_1 = -j400 = 400 \angle -90^\circ$

$Z_2 = 50 \angle 90^\circ$

$$Z_{OT} = \sqrt{(50 \angle 90)(400 \angle -90^\circ) + (400 \angle -90^\circ)^2}$$

$$= \sqrt{2000 - 40000} = 141.42 \angle -90^\circ$$

(For high filter (i.e. if C is in series arm) phase shift will be negative)

(c) $\frac{Z_1}{4Z_2} = \frac{-j200}{4 \times j100} = 0.5$ since $0 \geq \frac{1}{4Z_2} \geq -1$

$\therefore$ The frequency is within the pass band

(d) $\frac{Z_1}{4Z_2} = \frac{-j400}{4 \times j50} = -2$ since $\frac{Z_1}{4Z_2} < -1$

$\therefore$ The frequency is within the stop band

(e) $r = 2 \log_e \left\{\sqrt{1 + \frac{Z_1}{4Z_2}} + \sqrt{\frac{Z_1}{4Z_2}}\right\}$ or $\alpha + j\beta$

$= 2 \log_e \{\sqrt{1 - 0.5} + \sqrt{-0.5}\}$

$= 2 \log_e \{0.707 + j0.707\}$

$= 0 + j90X$

$\therefore \quad \alpha = 0$

$\beta = \frac{\pi}{2}$ rad

For half frequency $r \doteq 2 \log \{\sqrt{1 - 2} + \sqrt{-2}\}$

$\therefore \quad \alpha = 1.76$ and $\beta = -\pi$ rad

(f) From (a) & (b) We can say that the characteristics impedence in the pass band S pure resistive and in the attenuation band (i.e. Stop band) is pure reactive.

(9) Yes.

11.21 $R_K = \sqrt{\frac{L_{1K}}{C_{2K}}}$ or, $C_{2K} = \frac{L_{1K}}{R_K^2}$

$f_{0\pi} = \frac{1}{\pi \sqrt{L_{1KC2K}}}$

$\pi = \frac{1}{\pi \sqrt{L_{1K} \frac{L_{1K}}{R_K^2}}}$

$L_{1K} = \frac{R_K}{\pi f_{c1}}$

$= \frac{800}{\pi \times 10}$

$= 0.0255$ H

$C_{1K} = \frac{L_{1K}}{R_K^2}$

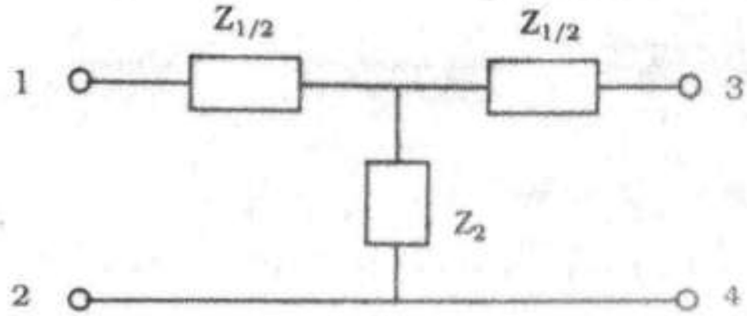
$= \frac{0.0255}{(800)^2}$

$= 3.98 \times 10^{-8}$ Farad

1000, 4000, 7000, 10,000 Hz are within pas band phase

Shift $\beta = \cos^{-1}\left(1 + \frac{Z_1}{2Z_2}\right)$

Now



Now, $Z_1 = j2\pi \times f \times 0.0255 = -j0.1602f$, Z_2

$= -j\frac{1}{2\pi f \times 0.04 \times 10^{-6}}$

$= -j3.98 \times 10^6$

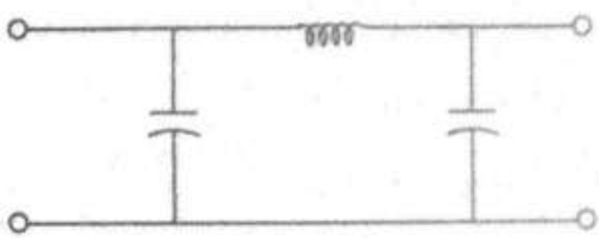
$\therefore \quad \frac{Z_1}{2Z_2} = \frac{j0.1602f}{2\left(\frac{-j3.98 \times 106}{f}\right)} = -2.013 \times 10^{-8} f^2$

$\therefore \quad \beta_{1000} = \cos^{-1}\{1 - 2.013 \times 10^{-8} \times 10^{6}\} = 11.5°$

$\beta_{400} = 47.32°$, $\beta_{7000} = 39.2°$, $\beta_{10,000} = 180°$

Attention $\alpha = \cos^{-1}\left(-1 - \frac{Z_1}{2Z_2}\right)$, find your self.

11.22



Here $Z_1 = jWL_1 = (2\pi f L_1) j = j \times 2\pi \times f \times 0.1$

$\therefore \quad Z_1 = j628f$

and $c_1 = \frac{c}{2}$

$\therefore \quad 2Z_2 = \frac{-j}{2\pi f c_1} = \frac{-j \times 10^6}{2\pi f \times 0.3} = \frac{-j530516.48}{f}$

$\therefore \quad Z_2 = \frac{-j265258.24}{f}$

Now, $\sqrt{\frac{Z_1}{4Z_2}} = \sqrt{5.92 \times 10^{-7} f^2 \angle 180^\circ} = 7.694 \times 10^{-4} f \angle 90^\circ$

$$\sqrt{1 + \frac{Z_1}{4Z_2}} = \sqrt{1 + 5.92 \times 10^{-7} f^2 \angle 180^\circ}$$

We know, $\alpha + j\beta = 2 \log_e \left(\sqrt{1 + \frac{Z_1}{4Z_2}} + \sqrt{\frac{Z_1}{4Z_2}} \right)$

$\therefore \alpha + j\beta = 2 \log_e (\sqrt{1 + 5.92 \times r^{-7} f^2} \angle 180^\circ + 7.694 \times 10^{-4} f \angle 90^\circ)$

At 500 Hz

$$\alpha + j\beta = 2 \log_e (\sqrt{1 + 5.92 \times 10^{-7} \times (500)^2 \angle 180^\circ} + 7.694 \times 10^{-4} \times 500 \angle 90^\circ$$

$\alpha + j\beta = 2 \log_e (0.923 + j0.3847)$

$\alpha = 0$ and $\beta = 0.79$ rad $= 45.26^\circ$

For others similar way.

11.23 (a) Decibel $= 10 \log_{10} \frac{W_{general}}{W_{reference}}$

$\Rightarrow \quad dB = 10 \log_{10} \frac{0.00001}{1 \times 10^{-3}} = -20 \quad \boxed{dB = 20}$

(b) $dB = 10 \log_{10} \frac{W_{general}}{W_{reference}} = 10 \log_{10} \frac{6}{1 \times 10^{-3}}$

$\boxed{dB = 37.78}$

11.24 Here $L_{1K} = 20H \; \frac{C_{2K}}{2} = 8\mu f \; \therefore C_{2K} = 16 \times 10^{-6} f$

We know for low pass filter—

$$f_c = \frac{1}{\pi \sqrt{L_{1K} C_{2K}}} = \frac{1}{\pi \sqrt{20 \times 16 \times 10^{-6}}} = 17.79 \text{ Hz}$$

$\therefore \quad f_c = 17.79$ Hz

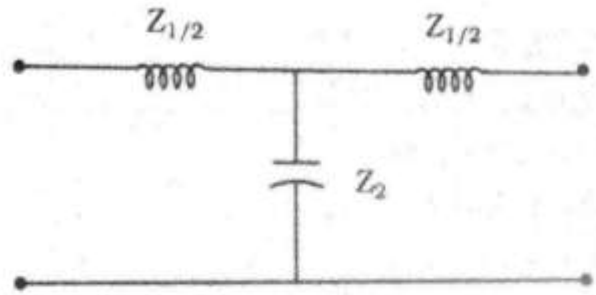
at 200 Hz

$$Z_{1K} = j2\pi \, L_{1K} f = j2\pi \times 200 \times 20 = 25132.74 \angle 90^\circ \text{ ohm}$$

$$2Z_{2K} = \frac{-j \times 10^6}{2\pi f \times 8} = 99.4 \angle -90^\circ \; Z_{2K} = 49.74 \angle -90^\circ$$

Now, $Z_{ork} = \sqrt{\frac{Z_{1k} Z_{2k}}{\left(1 + \frac{Z_{1k}}{4Z_{2k}}\right)}} = \sqrt{\frac{1250102.488 \angle 0}{1 + 126 \angle 180^\circ}} = 99.87j$

11.25



$Z_{1/2} = j100$

$Z_1 \quad = 200 \angle 90°$

$Z_2 \quad = -j1000$

$= 1000 \angle -90°$

$$\therefore \quad \frac{Z_1}{4Z_2} = \frac{200 \angle 90°}{4 \times 1000 \angle -90} = 0.05 \angle 180° = -0.05$$

$$Z_{OT} = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}$$

$= \sqrt{(200 \angle 90°)(1000 \angle -90°)(1 - 0.05)}$

$= 435.89 \angle 0°$

$$r = 2 \log_e \left\{\sqrt{1 + \frac{Z_1}{4Z_2}} + \sqrt{\frac{Z_1}{4Z_2}}\right\}$$

$\therefore \quad \alpha + j\beta = 2 \log_e \{\sqrt{0.05} + \sqrt{0.05}\}$

$= 2 \log_e \{0.97468 + j0.2236\}$

$= 2 \log_e \{1 \angle 12.92°\}$

$= 0 + j25.84°$

$\therefore \quad \alpha = 0, \beta = 25.84°$

(c) Since alternation zero

$\therefore$ The frequency is within the pass band.

(d) For 5 times frequency $Z_1 = 1000 \angle 90°$, $Z_2 = 200 \angle -90°$

$$\therefore \quad \frac{Z_1}{4Z_2} = \frac{1000 \angle 90°}{4 \times 200 \angle -90°} = 1.25 \angle 180 = 1.25$$

$\therefore \quad Z_{OT} = \sqrt{(1000 \angle 90^\circ)(200 \angle -90^\circ)\{1 - 1.25\}}$

$= 2 \log_e \{j0.5 + j1.110\}$

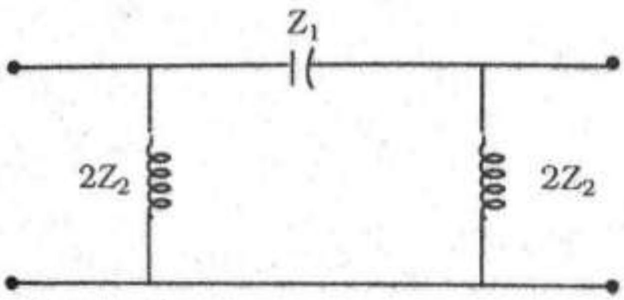
$= 2 \log_e \{1.618 \angle 90^\circ\}$

$= 0.96238 + j180^\circ$

$\therefore \quad \alpha = 0.96238$ Neper

Phase shift $\beta = \pi$ rad

11.26



$Z_1 = -j100 = 100 \angle -90^\circ$

$2Z_2 = j500$

$\therefore \quad Z_2 = 250 \angle 90^\circ = -0.1$

$$\frac{Z_1}{4Z_2} = \frac{100 \angle -90^\circ}{4 \times 250 \angle 90} = 0.1 \angle -180 = -0.1$$

(a) $Z_{o\pi} = \sqrt{Z_1 Z_2 / \left(1 + \frac{Z_1}{4Z_2}\right)}$

$= \sqrt{100 \angle -90^\circ (250 \angle 90^\circ)/(1 - 0.1)}$

$= 166.667 \angle 0^\circ$

(b) $r = \alpha + j\beta = 2 \text{ Log}_e \left\{\sqrt{1 + \frac{Z_1}{4Z_2}} + \sqrt{\frac{Z_1}{4Z_2}}\right\}$

$\therefore \quad \alpha + j\beta = 2 \log_e \{\sqrt{1 - 0.1} + \sqrt{-0.1}\}$

$= 2 \log_e \{0.94868 - j0.31628\}$

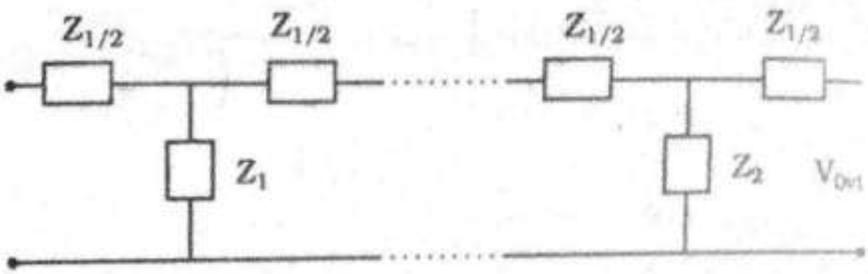
$= 2 \log_e \{1 \angle -18.435^\circ\}$

$= 0 - j36.87^\circ$

$\therefore \quad \alpha = 0, \beta = 36.87^\circ$

(c) Since alternation is zero. The reactances given are at pass band.

11.27



$$\alpha + j\beta = 2 \log_e \sqrt{\frac{Z_1}{4Z_2}} + \sqrt{\frac{Z_1}{4Z_2} + 1}$$

$$\frac{Z_1}{4Z_2} = \frac{j500 \times 2}{4 \times - j200} = - 1.25$$

$$\therefore \quad \alpha + j\beta = 2 \log_e \{\sqrt{-1.25} + \sqrt{-1.25 + 1}\}$$

$$= 2 \log_e \{1.118\, j + 0.5\, j\}$$

$$= 2 \log_e \{1.618 \angle 90^\circ\}$$

$$\alpha + j\beta = 2 \log_e 1.618 + j90^\circ$$

$$\therefore \quad \alpha = 0.962$$

$$\beta = 90^\circ$$

So, at ninth section = $\alpha^9 = 0.708$

So, output voltage = $(1 - 0.708) \times 100 = 29.18$ volt.

11.28

$$f_c = 5000 \text{ Hz}$$

$$R_k = 600 \text{ ohms}$$

So, $\sqrt{\frac{L_{2k}}{c_{1k}}} = R_k = 600$ ohm

$$\therefore \quad L_{2k} = 600^2 \times c_{1k}$$

and $c_{1k} = \frac{1}{4\pi \times 600 \times f_c}$

$$= \frac{1}{4\pi \times 600 \times 5000}$$

$$= 2.65 \times 10^{-8} f$$

So find L_{1k} yourself.

11.29 Do yourself

11.30 f_c = 5000 Hz

R_k = 600 oh

f_∞ = 4500 Hz

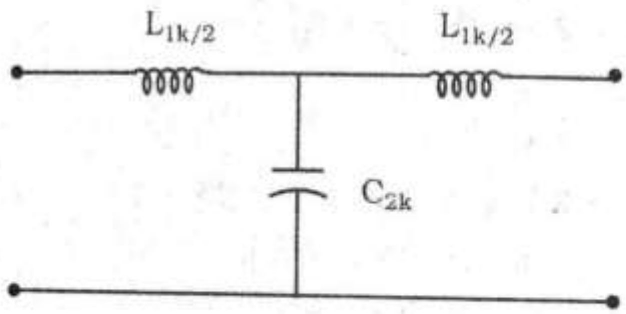
for low pass.

$$m = \sqrt{1 - \left(\frac{5000}{4500}\right)^2}$$

$$= 0.484\,j$$

and $L_{1k} C_{2k} = 600^2$

Find L_{1k} and C_{2k} from the above formulae then. $L_{1k/2}$ $L_{1k/2}$

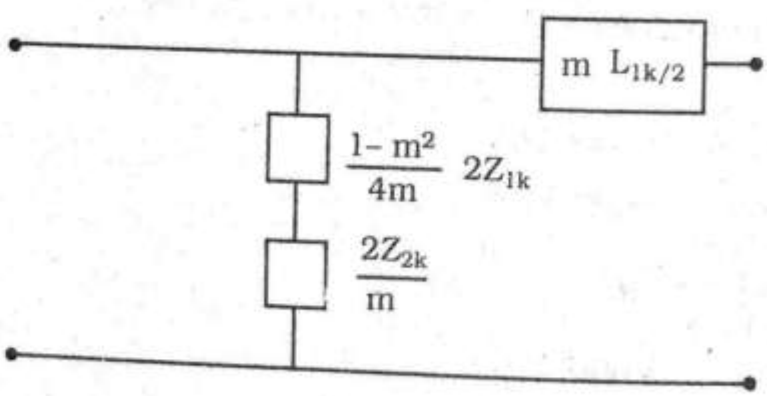


11.31 $L_{1k} = 0.30$ H

$c_{2k} = 0.03\ \mu f$

$m = 0.60$

$f_c = 800$ Hz



$$Z_{1k} = j2\pi \times 800 \times 0.3 = 1507.964j$$

$$Z_{2k} = \frac{1 \times 10^6}{2\pi \times 800 \times 0.03} = -6631.454\,j$$

So do yourself.

11.32 $Z_{1m} = m_{L1k} = L_{1m}$

$$Z_{2m} = \left(\frac{1 - m^2}{4m}\right) L_{1k} = L_{2m}$$

So in series $mc_{2k} = c_{2m}$

$$\therefore \quad f_c = \frac{1}{\pi \sqrt{L_{1k} c_{2k}}}$$

$$\text{R.H.S} = \frac{1}{\pi \sqrt{(L_{1m} + c_1 L_{2m})(c_{2m})}}$$

$$= \frac{1}{\pi \sqrt{(ML_{1k} + \frac{4(1 - m^2)}{4m} L_{1k})(mc_{2k})}} = \text{Do yourself}$$

33. Not important

Chapter–14

Transient Analysis

14.1 Graph ploting → Try yourself.

$$y = 311 \sin 377\, t$$

14.2 Transient current conection,

$$i = \frac{E_m}{z} \sin(\omega t + \lambda + \theta) \frac{-E_m}{z} \sin(\lambda - \theta)\, e^{-Rt/L}$$

if $L \to 0$ then $\qquad z = \sqrt{(\omega L)^2 + R^2}$

$$\theta = \tan^{-1}\frac{\omega L}{R} \qquad = R$$

or, $\quad \tan\theta = \dfrac{\omega L}{R}$

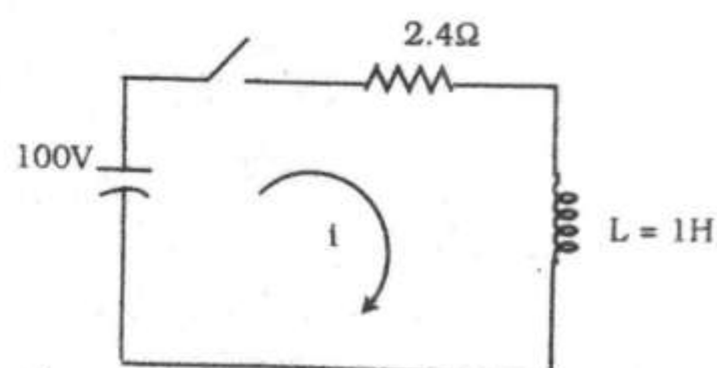
$$\text{so } \theta = 0 \; [\because L \to 0]$$

$$\therefore i \quad = \frac{Em}{R}\sin(\omega t + \lambda) - \frac{Em}{R}\lambda e^{-\infty}$$

steady state term

$$= \frac{E_m}{R}\sin(\omega t + \lambda)$$

14.3



$$0.4\, i \frac{di}{dt} = 10$$

or, $\quad \dfrac{di}{dt} + 0.4\, i = 10$

$\therefore$ The complete solution is $i = I_s + I_p = c_1 e^{-0.4t} + \dfrac{10}{0.4}$

$$= c_1 e^{-0.4t} + 25$$

At $t = 0, i = 0 \therefore 0 = c_1 + 25$

$\therefore \quad c_1 = -25$

$\therefore \quad i = -25e - 0.4t = 25$

$= 25(1 - e^{-0.4t})$

(a) at t = 1 sec, $i = 25(1 - e^{-0.4}) = 8.242$ amp

(b) at t = 2.5 sec $i = 25(1 - e^{-0.4 \times 2.5}) = 15.8$ amp

(c) $V_L = L\frac{di}{dt} = 1, -\frac{d}{dt}\{25(1 - e^{-0.4t})\} = 10e^{-0.4t}$

accelerating voltage at t = 1 sec is $10e^{-0.4} = 6.7$ volt

" " " t = 2.5 " " $10e^{-0.4 \times 2.5} = 3.68$ volt

(a) Here $L_1 = 1$ H, R = 0.4 ohm, t = 1 sec, E = 10 volt

We know that

$i = \frac{E}{R}(1 - e^{-0.4 \times 1}) = 8.2$ amp

(b) After 2.5 sec, $i = \frac{E}{R}(1 - e^{-Rt/L}) = \frac{10}{0.4}\left(1 - e^{\frac{-0.4 \times 2.5}{1}}\right)$

$\therefore \quad i = 15.802$ amp

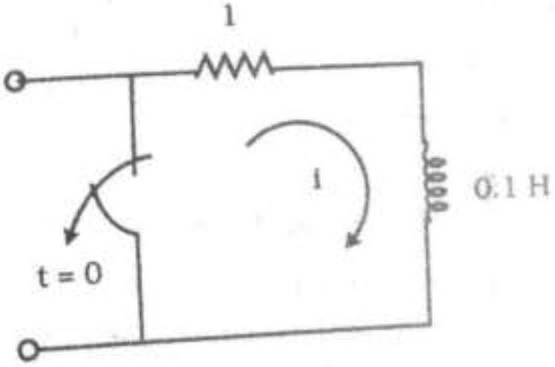
(c) After 1 sec, the value of voltage accelerating the current the resisturn = $R_i = 0.4 \times 8.242 = 3.2960$ volts.

Across the inductance $V_L = L\frac{di}{dt} = -L\left[-\frac{E}{R} - \frac{R}{L}e^{-Rt/L}\right]$

$= 6.703$ volts

$\therefore$ The accelerating voltage is, $E - L\frac{di}{dt} = -3.2968$ volts

14.4



$\therefore \quad i = 10e^{-10t}$

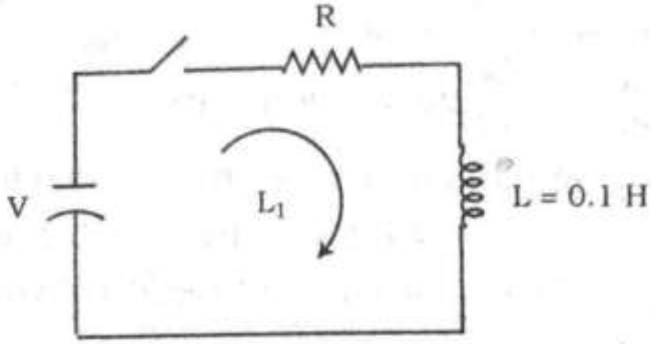
$\therefore$ After 0.1 sec, $i = 10e^{-10\times 1} = 3.68$ amp

Let the time be t_1

$\therefore$ $0.1 = 10e^{-10t}$ or $-10t = \ln(0.01) = -4.605$

$\therefore$ $t = 0.4605$ sec

14.5 Let the resistance be R



$\therefore$ $R_i + 0.1\frac{di}{dt} = V$ or, $\frac{di}{dt} + 10\,R_i = 10\,V$

at $t = 0, i = 0 \therefore c_1 = -v/R \therefore i = V/R\,(1 - e^{-10\,Rt})$

The final value of current = V/R after 2 sec, $i = 0.632\frac{V}{R}$

$\therefore$ $0.632\,V/R = V/R\,(1 - e^{-10\,R\times 2})$ or, $e^{-10R} = 1 - 0.632$

or, $-20\,R = \ln(0.368)$

$\therefore$ $R = 0.05$ ohm

14.6 $0.1\,i + \frac{di}{dt} = 10$ or, $\frac{di}{dt} + 0.1\,i = 10$

$\therefore$ $i = 4e^{-0.1\,t} + \frac{10}{0.1}$

$= 4e^{-0.1\,t} + 100$ at $t = 0, i = 0 \therefore 4 = -100$

$\therefore$ $i = 100\,(-e^{-0.1t})$

$V_L = L\frac{di}{dt} = 1\frac{d}{dt}\{100(1 - e^{-0.1t})\}$

$= 100 \times 0.1\,e^{-0.1t} = 10e^{-0.1t}$

$P_L = V_L i_L = 10e^{-0.1t} \times 100(1 - e^{-0.1t})$

$= 1000(e^{-0.1t} - e^{-0.2t})$

(a) At t = 10 sec, $i = 100(1 - e^{-0.1\times 10}) = 63.2$ amp

Stored energy = $V_2 \times Li^2 = \frac{1}{2} \times 1 \times (63.2)^2 = 1998$ Joule

(b) Energy dissipated $= iR^2 = R^2 - \frac{E}{R}(1 - e^{-Rt/L})$

$= ER(1 - e^{-Rt/L})$ Joule

$$O_R = \int_0^t i^2 R dt = R\int_0^t \frac{E^2}{R^2}(1 - e^{-Rt/L})^2 dt$$

$$= \frac{E^2}{R}\int_0^t (1 - e^{-Rt/L})^2 dt$$

$$= \frac{E^2}{R}\int_0^t (1 - 2.e^{-Rt/L} - 2Rt/L)\, dt$$

$$= \frac{E^2}{R}\left[t + \frac{2L}{R}e^{-Rt/L} - \frac{L}{2R}e^{-2Rt/L}\right]_0^t$$

$$= \frac{E^2}{R}\left[t - \frac{2L}{R}e^{-Rt/L} - Re^{-2Rt/L - 2L/R} + L/R\right] \text{ Joules}$$

14.7

$R_i + \frac{1}{c}\int i dt = V$ or, $R\frac{dq}{dt} + \frac{q}{c} = V$ or $1 \times 10^6 \frac{dv}{dt} + \frac{q}{50 \times 10^{-6}} = V$

or, $\frac{dq}{dt} + 0.02\, q = 10^{-6}$

$\therefore \quad q = c_1 e^{-0.02t} + \frac{10^{-6}}{0.02} V = 50 \times 10^{-6}\, V(1 - e^{-0.02t})$

Let the time be t_1

$\therefore \quad 0.632 \times 50 \times 10^{-6}\, r = 50 \times 10^{-6}\, V(1 - e^{-0.02t})$

or, $e^{-0.02t} = 1 - 0.632$

or, $-0.02t_1 = \ln(0.368)$

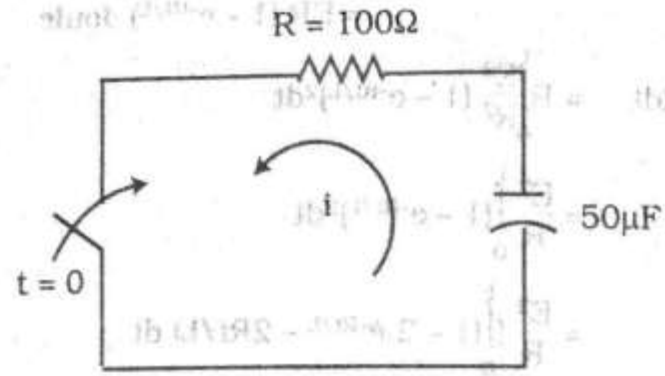
or, $t_1 = 50$ sec

(a) Let the time be t_1

$\therefore \quad 0.001 = 0.1e^{-20t1}$ or, $-20t_1 = \ln(0.01)$

$\therefore \quad t_1 = 0.23$ sec

(b) Initial voltage $V_c = \frac{0.1}{50 \times 10^{-6}}$



$R_i + \frac{1}{c} = 0$

or, $1000\frac{dq}{dt} + \frac{q}{50 \times 10^{-6}} = 0$

or, $\frac{dq}{dt} + 20q = 0$

$\therefore \quad q = c_1 e^{-20t}$ coumb

$t = 0, \; q_0 = 0.1$ c

$\therefore \quad 0.1 = c_1 \; \therefore \; q = 0.1e^{-20t}$

$\therefore \quad i_c = \frac{2000}{1000} = 2$ amp

(c) Then

$V_0 = \frac{0.001}{50 \times 10^{-6}}$

$= 20$ V

$\therefore \quad i = \frac{20}{1000} = 0.02$ amp

14.9 Soln : $Q_0 = 0.1$ coulomb $c = 100 \times 10^{-6} f$, $R = 10$ k ohm

Decay of charge in 1 sec, $q = CE\, e^{-t/RC}$

$$q = 100 \times 10^{-6} \times \frac{0.1}{100 \times 10^{-6}} \times e^{-\frac{1}{10{,}000 \times 100 \times 10^{-6}}}$$

$= 0.0367$ columb

We know, $W_c = \int_0^{\infty} p_0 dt = \int_0^{\infty} V_c\, i dt = \int_0^{\infty} \frac{q}{c} \cdot \frac{dq}{dt} \cdot dt$

$= \int_0^{\infty} \frac{q}{c} dq = \frac{1}{2}\frac{q^2}{c}$ But here

$$W_c = \frac{(Q_0 - q)^2}{2c} = \frac{(0.1 - 0.0367)^2}{2 \times 100 \times 10^{-6}} = 20 \text{ Joules}$$

14.10 Soln :

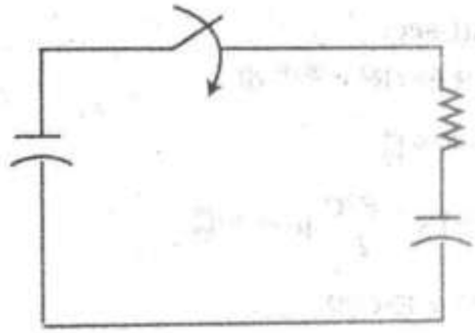
$R = 10.000$ ohm

$c = 100 \times 10^{-6} f$

$t = 1$ sec

$w_c = 19.98$ Joule in 1 sec

$Q_0 = 0, E = ?$



We know,

$$W_c = \int_0^\infty p_0 dt = \int_0^\infty V_c i dt + \int_0^\infty \frac{q}{C}\frac{dq}{dt} \cdot dt$$

$$W_c = \int_0^\infty \frac{q}{C} dq = \frac{q^2}{2c}$$

$$\therefore \quad 19.98 = \frac{q^2}{100 \times 10^{-6}} = 632.133 \text{ volts}$$

14.11 (a)

Here given,

$R = 1 \times 10^6$

$C = 1 \times 10^6 f$

$E = 100$ volts

$$q = CE - (CE - Q)_e^{-t/RC}$$

$$= 10^{-6} \times 100\,(1 - e^{-1/106 \times 10^{-6}})$$

$$= 10^{-4} \times (1 - e^{-1}) = 6.32 \times 10^{-5}$$

Energy stored,

$$E_S = \frac{q^2}{2c} = \frac{(6.32 \times 10^{-5})^2}{2 \times 10^{-5}}$$

$$= 1.99712 \times 10^{-3} \text{ watt/sec}$$

(b) $E_R = \int_0^t i^2 R dt = \int_0^t R.\frac{E^2}{R}.\ e^{-2t/Rc}dt$

$= \frac{E^2}{R}\int_0^t e^{-2t/Rc}dt = \frac{E^2}{R} \times \frac{R_c}{2}\ [e^{-2t/Rc}]^t$

$= \frac{-E^2c}{2}[e^{-2t/RC}] + \frac{E^2C}{2}$

$= \frac{100^2 \times 10^{-6}\left[0 - \frac{20 \times 1}{10^6 \times 10^6}\right]}{2} + \frac{100^2 \times 10^{-6}}{2}$

$= 4323 \times 10^{-3}$ watt-sec

(c) E_R full $= \int_0^\infty i^2 R dt = \int_0^\infty R.E^2/R^2\ e^{-2t/Rc}dt$

$= \frac{E^2}{R} \times \frac{-R_c}{Z}[e^{-2t/Rc}]_0^\infty$

$= \frac{-E^2C}{2}[e^{-2t/Rc}] = \frac{-E^2C}{2}[e^{-2t/Rc}]_0^\infty$

$= \frac{E^2C}{2}e^{-2 \times 0/Rc} = E^2C/2$

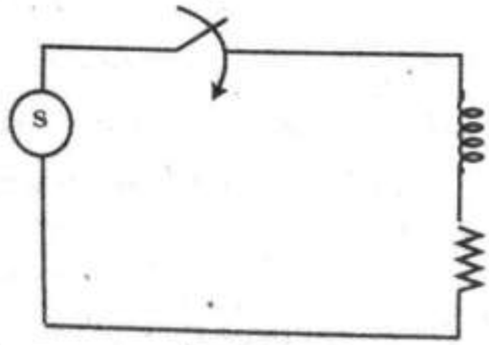
$= \frac{100^2 \times 10^{-6}}{2} = 5 \times 10^{-3}$ watt-sec

14.12 Do yoursef

14.13 We know, $i = \underbrace{\frac{E_m}{Z}\sin(Wt + \lambda - \theta)}_{\text{Steady state}} - \underbrace{\frac{E_m}{Z}\sin(\lambda - \theta)e^{-Rt/L}}_{\text{Transient state}}$

Here, $E_m = 100$, $wt = 377t$, $\lambda = \frac{\tau}{4}$

$\theta = \tan^{-1}\frac{WL}{R} = \tan^{-1}\frac{377 \times 1}{1} = 89.848°$



$L = 1$ H, $R = 1$ oh, $Z = \sqrt{R^2 + (WL)^2}$

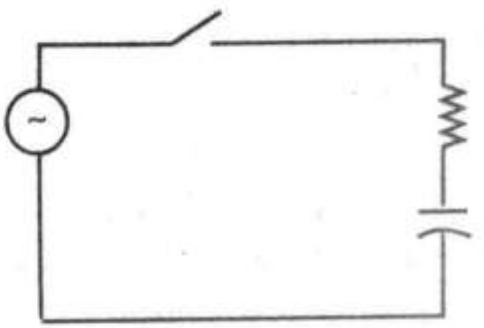
$= \sqrt{1^2 + (377 \times 1)^2} = 377$ ohm

When, $t = 0$

$$i_{ss} = \frac{100}{377} \sin(\pi/4 - 89.848) = -0.187 \text{ amp}$$

$$i_{tr} = \frac{-100}{377} \sin(\pi/4 - 89.848) = 0.187 \text{ amp}$$

14.14 We know,



$$q = \frac{-E_m}{W\sqrt{R^2 + X_c^2}} = \cos(\omega t + \lambda t + \theta) + \frac{E_m e^{-t/Rc}}{\omega\sqrt{R^2 t \times c^2}} \times \cos(\pi + \theta)$$

$$q = \frac{-100}{377\sqrt{1^2 + \left(\frac{1}{377 \times 100 \times 10^{-6}}\right)^2}}$$

$$\cos\left(30^\circ + \tan^{-1}\frac{1}{377 \times 100 \times 10^{-6}}\right)$$

$$+ \frac{100 \times e^0}{377\sqrt{1^2 + \left(\frac{1}{377 \times 100 \times 10^{-6}}\right)^2}}$$

$$\cos\left(30^\circ + \tan^{-1}\frac{1}{377 \times 100 \times 10^{-6}}\right)$$

$$= 0.00499 - 0.00499$$

$$\therefore \quad q_{ss} = 0.00499$$

$$q_{tr} = -0.00499 \text{ coulb}$$

(b) $i = \frac{E_m}{Z} \sin(\omega t + \lambda + \theta) \frac{-E_m}{WCRZ} \cos(\lambda + \theta)\, e^{-t/Rc}$

When $t = 0$, $z = \sqrt{I^2 + \left(\frac{1}{77 \times 100 \times 10^{-6}}\right)^2} = 26.544$

$$i = \frac{100}{26.544} \sin(30 + 87.84) - \frac{100}{26.544} \cos(30 + 87.848) \times \frac{C^0}{377 \times 100 \times 10^{-6}}$$

$= 3.33 + 46.67$

$i_{ss} = 3.33$

and $i_{tr} = 46.67$ amp

14.15 $R = 100$ ohm, $L = 0.1$ Henry

$C = 200 \times 10^{-6}$ $Q_0 = 0$

$$a = \frac{R}{21} = 500$$

$$b = \sqrt{\left(\frac{R}{2L}\right)^1 - \frac{1}{LC}}$$

$= 447.213$

$\therefore (-a + b) = 477.213 - 500$

$= -52.787$

$\therefore \quad (-a + b)t = e^{-52.787 \times 0.01} = 0.589$

$(a - b) = -947.213$

$\therefore \quad e^{(-a-b)t} = e^{-947.213 \times 0.01} = 7.696 \times 10^{-5}$

$\therefore \quad [e^{(-a+b)t} - e^{(-a-b)t}] = 0.588$

$CE = 200 \times 10^{-6} \times 50 = 0.01$

$RC = 100 \times 200 \times 10^{-6} = 0.02$

$\sqrt{R^2C^2 - 4LC} = 0.0178$

We know that

$$i = \frac{CE - Q_0}{R^2C^2 - 4LC}$$

$$\left[\frac{R_c + \sqrt{R^2C^2 - 41C}}{2\sqrt{R^2C^2 - 41C}} e^{(-a+b)t} - \frac{RC - R^2C^2 - 4LC}{2\sqrt{R^2C^2 - 4LC}} e^{(-a-b)t}\right.$$

$= 0.01 - 0.01$

$$\left[\frac{0.02 + 0.0178}{2 \times 0.0178} \times 0.589 - \frac{0.02 - 0.0178}{2 \times 0.0178} \times 7.696 \times 10^{-5}\right]$$

$= 0.01 - 0.01\ [0.625 - 4.75 \times 10^{-6}]$

$= 0.00375$ coulomb

14.16 According to Sir (Shaidulla).

$R = 4$ ohm. $L = 0.1$ H. $C = 200 \times 10^{-r}$, $t = 0.01$ sec

$E = 1000$ V. $Q_0 = 0$

$$A = \frac{R}{2L} = \frac{5}{2 \times 0.1} = 25, \; B = \sqrt{\frac{1}{LC} - \frac{RL}{4L}} = 22.204$$

$$\sqrt{4 \times 0.1 \times 200 \times 10^{-6} - (5 \times 200 \times 10^{-6})}$$

$$= 0.00888$$

(a) $$i = \frac{2(CE - Q_0)}{\sqrt{4LC - R^2C^2}} \sin Bt = \frac{2\ CEe^{-4t}}{4_{LC} - R^2C^2} \sin Bt$$

$$= \frac{2 \times 200 \times 10^{-6} \times 1000e^{-25 \times 0.01}}{0.00888} \sin(222.204 \times 0.01)$$

$$= \frac{0.311}{0.00888} \times 0.769$$

$= 35 \times 0.796 = 27.87$ Ans.

(b) Yes

(c) $$\pi = \frac{\beta}{2\pi} = \frac{222.204}{2\pi} = 35.36 \text{ Hz}$$

$$q = \frac{CE - 2(CE - \theta_0)Lce^{-t}}{\sqrt{4LC - R^2C^2}} \sin(Bt + \theta)$$

$$\varphi = \tan^{-1} \frac{\sqrt{4LC - R^2C^2}}{RC}$$

$$= \tan^{-1} \frac{0.00888}{5 \times 200 \times 10^{-6}} = 83.57^\circ$$

$$q = \frac{200 \times 10^{-6} \times 1000 - 2 \times 200 \times 10^{-6} \times 1000\sqrt{0.1 \times 200 \times 10^{-6}}}{0.0888}$$

$$\times e^{-25 \times 0.01} \sin(222.204 \times 0.01) + 1.45)$$

$$= 0.2 - \frac{1.39 \times 10^{-3}}{0.00888} \sin(3.67204)$$

$$= 0.2 - 0.1565 \sin\left(3.67204 \times \frac{180}{\pi}\right)$$

$= 0.279$ coloumb

14.18 Given,

$e = 141.\sin(377t - 45°)$ volts

$R = 1$ ohm $\qquad F_m = 141.0$ volt

$L = 0.041$ A $\qquad \beta = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$ = Do yourself

$c = 18.7\mu f$

$Q_0 = 0$

(a) $i = k_1 \sin(k_2 t + k_3) + k_4 e^{5t} \sin(k_6 t - k_7)$ amp

Where, $i = \frac{E_m}{z} \sin(\omega t + \lambda - \partial) + I_t\, e^{-at} \sin(\beta t - 6)$

$$I_t = \sqrt{\left[\frac{E_d}{\beta L}\right]^2 + \left[\frac{E_m}{z} \sin(\lambda - \theta)\right]^2}$$

$$6 = \tan^{-1} \frac{E_m \beta L \sin(\lambda - \partial)}{E_d z}$$

Here,

$K_1 = \frac{E_m}{z}$ $\qquad I_t = K_4$

$a = 5$

$K_3 = \lambda\theta$

$K_6 = \beta$

$K_2 = K_2$

$K_7 = 6$

Find the corresponding values hence write the equation.

(b) Sketch yourself.